Q1 APPLICATION OF FUSION REACTION (10 pts)

Fusion reaction, where two nuclei are combined to form a heavier nucleus, is essential in understanding not only stellar mechanics but also for achieving controlled nuclear fusion as a clean and sustainable energy source on Earth. Among the nuclear reactions studied for these purposes, the Deuterium-Deuterium (D-D) and Deuterium-Tritium (D-T) neutron-producing fusion reactions are particularly significant. Only these two reactions will be considered throughout this problem.

Part 1. Energy from Fusion Reaction (2.0 pts)

Consider the following nuclear particles:

Table 1. *Nuclear particles involved in fusion reaction*

Particles	Symbol	Mass (u)
Deuterium (D)	${}_{1}^{2}\mathrm{H}$	2.014102
Tritium (T)	${}_{1}^{3}\mathrm{H}$	3.016049
Helium-3	${}_{2}^{3}\mathrm{He}$	3.016029
Helium-4	${}_{2}^{4}\mathrm{He}$	4.002603
Neutron	${}_{0}^{1}\mathrm{n}$	1.008665

1.1 **(a)** Calculate the Q-values in MeV for the D-D and D-T nuclear fusion reactions. **0.2 pt**

(b) The Q-value or the energy released from the reaction is split into the kinetic energies (E) of the reacting products. Derive an expression for the kinetic energy of the neutron (E_n) in terms of the masses of the reaction products (m_n and m_x) under the general reaction (reactants$\rightarrow X + n$) and the corresponding Q-value. Assume that the initial momentum is zero. **0.6 pt**

(c) From the expression obtained from (b), calculate the kinetic energy of the neutrons produced from the D-D and D-T reactions. **0.2 pt**

Fusion reaction was discovered in 1921 by Arthur Eddington, 17 years before the discovery of fission. However, while fission-based nuclear power plants are now supplying ~10% of the world's electricity since becoming commercially feasible by 1957, fusion reactors have yet to demonstrate their feasibility. Nonetheless, research and development in fusion technology is continued to ensure alternative clean energy for the future. The following question will compare the energy that can be derived from fusion and fission nuclear reactions with the energy derived from coal, which currently supplies ~36% of the world's electricity.

1.2 In 2022, it was reported that the worldwide average energy consumption per person is $3,600\ \mathrm{kWh}$.

(a) Assume that the reaction energy is fully recoverable for a fusion plant with an efficiency of 35%, how many kg of D-T reactants will be needed to provide $3,600\ \mathrm{kWh}$? How about D-D reactants? **0.5 pt**

(b) The average energy released per fission reaction is $200\ \mathrm{MeV}$. Assuming that this energy is fully recoverable, and that the fission plant has an efficiency of 30%, how many kg of fission reactants are needed to supply the same amount of energy? Note that the mass of $^{235}\mathrm{U}$ is $235.044\ \mathrm{u}$. **0.25 pt**

(c) How many kg of coal is needed to provide the same amount of energy if combustion of bituminous coal releases $31\ \mathrm{kJ\ per\ g}$ of fuel? **0.15 pt**

(d) What is the ratio of your answer for the mass of coal and the mass of D-T reactants from your answer in 1.2 (a)? **0.1 pt**

Part 2. Tritium Production (2.0 pts)

D-T fusion reaction is considered more feasible for fusion energy research due to its higher interaction probability and higher energy released per reaction compared to D-D fusion. Moreover, tritium, though not naturally abundant, can be bred from lithium. Lithium is more readily available than deuterium, which is the sole source for the D-D reaction. Natural lithium is 4.85% ^{6}Li and 95.15% ^{7}Li. Although tritium $^{3}_{1}H$ can be produced from both isotopes, the following ^{7}Li reaction is a threshold reaction, which requires fast neutrons. (Note: Percentages are isotopic abundances.)

$$^{7}_{3}Li + ^{1}_{0}n \rightarrow ^{4}_{2}He + ^{3}_{1}H + ^{1}_{0}n \quad (1)$$

Meanwhile, ^{6}Li is a strong neutron absorber which can produce tritium through the following (n, α) reaction with interaction cross section of $\sigma = 940 \text{ barn}$.

$$^{6}_{3}Li + ^{1}_{0}n \rightarrow ^{4}_{2}He + ^{3}_{1}H \quad (2)$$

The total amount of $^{3}_{1}H$ that can be produced from reaction (2) can be calculated from the reaction rate R:

$$R = \sigma N \phi V \quad (3)$$

where ϕ is the neutron flux and V is the volume of the sample. Meanwhile the nuclide density (N) is shown below, which is expressed in terms of density (ρ), Avogadro's number (N_A) and atomic mass($m_{^6Li} = 6.015122 \text{ u}$):

$$N = \rho N_A / m \quad (4)$$

2.1 Calculate the minimum incident neutron energy, in MeV, that is required to produce $^{3}_{1}H$ from ^{7}Li. Note that $m_{Li7} = 7.016003 \text{ u}$. Express the answer with 4 significant figures. **0.8 pt**

2.2 **(a)** Calculate the mass (in g) of $^{3}_{1}H$ nuclides produced from 1 kg of natural lithium that is irradiated for 24 h in a fission reactor with a neutron flux of $3.5 \times 10^{14} \text{ n cm}^{-2} \text{ s}^{-1}$. Assume that the irradiator source consists of purely thermal neutron. **1.0 pt**

(b) A D-T fusion reactor with $1,000 \text{ MW}$ output will require approximately 150 kg of tritium fuel per year. If we can only irradiate natural lithium with thermal neutron once in a reactor for 24 h, how much ^{6}Li should we place inside to produce 150 kg of tritium required for annual operation? **0.2 pt**

Part 3. Overcoming the Coulomb barrier (2.5 pts)

Electric potential energy (U) is the total work done to bring a system of charges into a certain configuration.

3.1	Calculate the energy required to bring a deuteron at contact to a target: **(a)** deuteron for D-D reaction, and **(b)** triton for D-T reaction.	**1.6 pts**

Due to the positive charge of the nucleus, fusion reaction between light nuclei occurs only when the reacting particles are sufficiently energetic to overcome the Coulomb repulsive force. This requires heating the reactants to very high temperatures to attain the plasma state. The energy distribution of fusion reactants in plasma state is given by the Maxwell-Boltzmann distribution law, which relates the plasma temperature (T) to the average energy $\left(\overline{E}\right)$ of the particles as follows:

$$\overline{E} = \frac{3k_BT}{2} \tag{5}$$

where k_B is the Boltzmann constant. We assume that the particles undergo elastic collision. The actual ignition temperature for D-D and D-T reactions is around $4.5 \times 10^8\ \mathrm{K}$ and $1.5 \times 10^8\ \mathrm{K}$ respectively. It differs from the theoretical ignition temperature due to quantum tunneling and the wide range of energy of the reacting nuclei, which is described by the Maxwell-Boltzmann distribution.

3.2	**(a)** Determine the corresponding temperatures for the energies calculated for D-D and D-T reactions from 3.1.	**0.6 pt**
	(b) Calculate the percentage difference between the actual and theoretical ignition temperatures for the D-D and D-T reactions. Use the following formula for % difference: $\%diff = \frac{\lvert X_1 - X_2 \rvert}{\left(\frac{X_1 + X_2}{2}\right)} \times 100\%.$	**0.3 pt**

Part 4. Portable Neutron Generators (3.5 pts)

Although power reactors based on fusion reaction are not yet commercially available due to the very high temperature requirements for the reacting particles, D-D and D-T fusion reactions have been used in portable neutron generators (NG) for research and industrial applications. An important parameter for NG applications is the total neutron yield S:

$$S = N_s \sigma \varphi \tag{6}$$

where N_S is the surface density or the number of target nuclei per unit area of the target, σ is the reaction cross section, and φ is the incident particle rate, which can be expressed empirically in terms of the deuteron beam current (I) in amperes and the incident particle charge (q):

$$\varphi = \frac{I}{q} \bullet a \tag{7}$$

In equation 7, a is an empirically determined unitless parameter that is dependent on the configuration of a particular neutron generator unit. For an incident deuteron with $100\ \mathrm{keV}$ energy, the cross section for the D-D reaction is $16.9\ \mathrm{millibarns}$ while the corresponding cross section for the D-T reaction is 293 times higher.

Consider a $1\ \mathrm{mm}$ thick titanium layer that was subjected to hydridation until the titanium to hydrogen ratio reached 1:1.64. The characteristics of the titanium hydrides are summarized in **Table 2**.

Table 2. *Characteristics of hydridated titanium targets*

TiH species	Density, ρ	Molecular Mass
$TiD_{1.64}$	$3.92\ \mathrm{g\ cm^{-3}}$	$51.170127\ \mathrm{u}$
$TiT_{1.64}$	$4.03\ \mathrm{g\ cm^{-3}}$	$52.813320\ \mathrm{u}$

4.1	Calculate the surface density N_s of the D and T atoms of **(a)** $TiD_{1.64}$ and **(b)** $TiT_{1.64}$ respectively. Note that $N_s = N \cdot l$ where N is the atom density and l is the sample thickness.	**1.4 pt**

4.2	A particular neutron generator unit with targets as described in this problem was determined to have the parameter value $a = 6.25 \times 10^{-4}$. Calculate the required beam current (in amperes) for this $100\ \mathrm{keV}$ neutron generator to produce a neutron yield of $S = 1 \times 10^{7}\ \mathrm{n\ s^{-1}}$ from D-D reactions and D-T reactions.	**1.4 pt**

One application of the portable neutron generator is neutron radiography (NR), which uses a neutron beam to visualize the internal structures of materials. This technique is highly effective for materials with low atomic numbers. The aperture in the neutron generator where the generated neutrons exit is commonly referred to as the neutron exit port. For a neutron generator with an isotropic neutron yield (S), the flux (ϕ) at a distance which is significantly larger than the neutron exit port (r), can be approximated by:

$$\phi = \frac{S}{4\pi r^2} \tag{8}$$

For this neutron flux to reach an object to be radiographed, the NR system uses a collimator to guide the neutrons towards the object. The collimator typically resembles a truncated cone or pyramid and may have a circular, square, or rectangular cross-section. The emerging flux from the collimator outlet aperture (ϕ) can be estimated using the relation:

$$\phi = \frac{\phi_0 A}{4\pi L^2} \tag{9}$$

where ϕ_0 is the flux from the collimator inlet aperture, A is the area of the collimator inlet aperture, and L is the distance from the collimator inlet to the imaging plane.

4.3	**(a)** Calculate the ϕ if the inlet aperture of the collimator is $3\ \mathrm{cm}$ away from the neutron generator, as described in 4.2. Assume that the neutron exit port is significantly smaller than $3\ \mathrm{cm}$.	**0.2 pt**
	(b) Consider a circular collimator that initially has a diameter of D_1 with the corresponding ϕ_1. Suppose the diameter of this collimator is adjusted to D_2 while keeping the ϕ_0 constant. Write the expression for the flux ϕ_2 in terms of ϕ_1.	**0.5 pt**

Q2 STERILE INSECT TECHNIQUE (SIT) FOR MOSQUITOES (10 pts)

Scientists combating mosquito-borne diseases such as malaria, dengue, West Nile virus, chikungunya, yellow fever and Zika are developing the use of the Sterile Insect Technique (SIT). This technique involves the mass breeding, sterilization, and release of sterile male insects into the field to mate with their female counterparts. The resulting offspring from this mating are not viable, thus reducing the population of the next generation through continuous release. In SIT, insects are rendered sterile by exposure to ionizing radiation such as gamma rays. Cobalt-60 or ^{60}Co can be used for this irradiation, which sterilizes without inducing radioactivity. It is essential to select a dose that effectively sterilizes male mosquitoes without significantly impacting their lifespan and mating competitiveness.

Part 1. Absorbed Dose Measurement using Fricke Dosimeter (4.5 pts)

In SIT, the irradiation dose delivered via ^{60}Co gamma rays plays a crucial role. The validation and routine control of this sterilization process relies on dosimetry, which is the measurement of the radiation dose received by the insects. Among the different types of dosimeters, Fricke dosimeter, commonly known as ferrous sulfate dosimeter, is one of the most versatile chemical dosimeters available. It works by tracking how ionizing radiation oxidizes ferrous ions $\left(Fe^{2+}\right)$ to ferric ions $\left(Fe^{3+}\right)$. The concentration of these ferric ions, which increases with radiation exposure, can be measured spectrophotometrically at $303\ \mathrm{nm}$. The Fricke dosimeter will be used to determine the absorbed dose of the mosquito pupae in a Petri dish.

The SI unit of absorbed dose is gray (Gy), which is defined as “the absorption of 1 joule of energy by 1 kilogram of matter”. Suppose a radioactive material has an activity A, and each disintegration releases a gamma ray photon with an energy E.

1.1	Write an equation for the power (total energy released per unit time) from gamma rays, in $\mathrm{J\,h^{-1}}$, if A is reported in MBq, and E is expressed in MeV $\left(1\ \mathrm{J} = 6.2415 \times 10^{18}\ \mathrm{eV}\right)$.	**0.5 pt**

1.2	Write an equation for the power density per unit area, in units of $\mathrm{J\,h^{-1}\,m^{-2}}$, at a distance r from the radioactive material source. Assume an isotropic gamma emission.	**1.0 pt**

As the gamma ray photons travel from the source to the Fricke dosimeter positioned at a distance r meters away, they can be absorbed by the air. The attenuation due to this absorption of gamma ray photons per unit distance travelled to the Fricke dosimeter is given by the equation:

$$I = I_o e^{-\mu_a r} \tag{1}$$

where I_o is the initial gamma ray photon intensity, I is the gamma ray photon intensity after travelling a distance r, and μ_a is the linear attenuation coefficient for air, in m^{-1}.

The dimensionless quantity $e^{-\mu_a r}$ is the attenuation factor f; it gives the fraction of the initial gamma ray photons that travelled the distance r without being absorbed. An alternative form is the mass attenuation coefficient, $\frac{\mu}{\rho}$, which is the linear attenuation coefficient μ normalized to the density ρ of the material. We can provide a more general expression for the attenuation factor f for gamma ray photons that travelled a distance x meters through arbitrary matter:

$$f = e^{\left(-\left(\frac{\mu}{\rho}\right)\rho x\right)} \tag{2}$$

Assuming all attenuated photons' energies are fully absorbed by the material, the dose rate (DR, with units of $\mathrm{Gy\ h^{-1}}$) at a distance r from the source is given by:

$$DR\ (\mathrm{Gy\ h^{-1}}) = \frac{1.442\times10^{-4}AE}{\pi r^2}\left(\frac{\mu}{\rho}\right) \tag{3}$$

where A is expressed in MBq and E is in MeV.

1.3	Calculate the dose rate (in $\mathrm{Gy\ h^{-1}}$) and dose (in Gy) to the Fricke dosimeter positioned $50\ \mathrm{cm}$ away from a ^{60}Co source with an activity of $8000\ \mathrm{Ci}$ after exposure for 10 minutes. ^{60}Co produces gamma rays with an average energy of $1.25\ \mathrm{MeV}$. The mass attenuation coefficient for a standard Fricke (ICRU 44) is $0.072\ \mathrm{cm^2\ g^{-1}}$.	**1.5 pt**

Ionizing radiation oxidizes Fe^{2+} to Fe^{3+}, and the radiation chemical yield $G\left(Fe^{3+}\right)$ was found to be proportional to the absorbed dose. The increase in the concentration of Fe^{3+}, compared to the concentration in the unirradiated solutions measured on the same day, is calculated from the increase in the optical density (OD) at $303\ \text{nm}$ at $25\ ^{\circ}\text{C}$. The absorbed dose to the Fricke dosimeter which was exposed to ionizing radiation is given by:

$$D = \frac{\Delta OD}{\varepsilon G(Fe^{3+})\rho l} \quad (4)$$

where ΔOD is the increase in optical density, ε is the extinction coefficient of Fe^{3+} minus the extinction coefficient of Fe^{2+} at $303\ \text{nm}$, $G\left(Fe^{3+}\right)$ is the radiation chemical yield of Fe^{3+}, ρ is the density of the Fricke solution, and l is the length of the light path of the cuvette.

1.4	Using the exposed Fricke dosimeter described in 1.3, determine the radiation chemical yield for Fe^{3+} in $\mu\text{mol J}^{-1}$. The optical density measured at $303\ \text{nm}$ at $25\ ^{\circ}C$ was 0.1840 after the completion of irradiation and 0.0030 before irradiation. $\varepsilon = 2174\ \text{L mol}^{-1}\,\text{cm}^{-1}$, $\rho = 1.024\ \text{g cm}^{-3}$, $l = 1\ \text{cm}$.	**0.5 pt**

1.5	In a hypothetical scenario where each ^{60}Co gamma ray photon deposits $300\ \text{keV}$ of energy into a $1\ \text{g}$ Fricke dosimeter per interaction, how many photon interactions would be needed for the dosimeter to receive absorbed doses of $10\ \text{Gy}$, $20\ \text{Gy}$, $30\ \text{Gy}$, $40\ \text{Gy}$, and $50\ \text{Gy}$? These are doses that have been evaluated for use in the SIT for mosquitoes.	**1.0 pt**

Part 2. Egg Hatch and Dose in SIT (3.5 pts)

In the SIT, the egg hatch percentage plays a crucial role. This percentage represents the proportion of eggs that successfully develop into larvae and hence directly impacts the mosquito population. By increasing the irradiation dose, a lower egg hatch percentage can be achieved. The continuous release of sterile males results in fewer mosquitoes in subsequent generations. **Figure 1** represents a relationship between egg hatch percentage and irradiation dose.

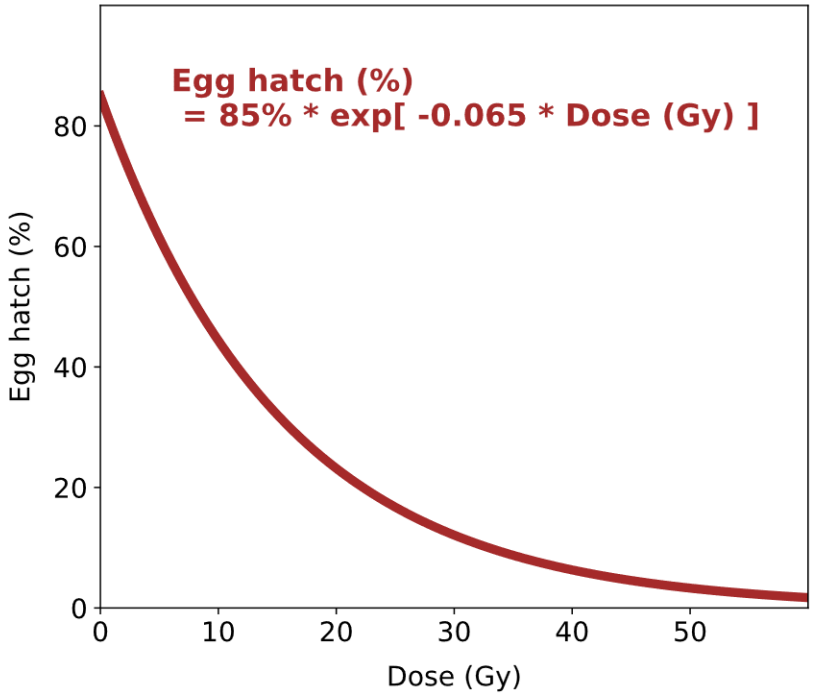

Figure 1. *Relationship between irradiation dose and egg hatch (%)*

A bare ^{60}Co source was used to irradiate a Petri dish of male mosquito pupae at a certain dose. However, laboratory tests revealed that the egg hatch of their offspring was only 40%. The calculated relationship between the egg hatch and the irradiation dose in Gy was:

$$Egg\ hatch\ (\%) = 85\% \times exp(-0.065\ Dose) \quad (5)$$

2.1	Based on equation 5 and the observed egg hatch, calculate the dose that was used to irradiate the mosquito pupae.	**0.5 pt**

2.2	If the irradiation dose that resulted in a 40% egg hatch was administered over a 10-minute period, what should have been the duration of irradiation to achieve a 1% egg hatch?	**1.0 pt**

The introduction of a new ^{60}Co source increased the dose rate, leading to a fixed one-minute irradiation period. This change resulted in a higher dose, which effectively reduced the egg hatch rate to 0.1%. However, this higher dose had a detrimental effect on the sterile male mosquitoes, compromising their mating competitiveness, a key factor for SIT's success. To maintain effective population control while ensuring the competitiveness of the sterile males, a lead shield is proposed with the aim of slightly increasing the egg hatch rate to 2%. This rate, while higher than 0.1%, still allows for effective population control, assuming the males remain competitive. The proposed lead shield could play a crucial role in fine-tuning the irradiation dose while preserving the quality and competitiveness of the sterile males.

The shielded dose can be calculated by the equation below:

$$D_i = D_0 exp(-(\mu/\rho)\rho x) \quad (6)$$

where D_i is the shielded dose in Gy, D_0 is the unshielded dose, (μ/ρ) is the mass attenuation coefficient of the material, ρ is the density of the material, and x is the thickness of the material.

2.3 Assuming no buildup factor effects from the proposed lead shield and that ^{60}Co gamma rays are monoenergetic at $1.25\ \mathrm{MeV}$, calculate the required lead thickness (in cm) to increase the egg hatch from 0.1% to 2%. Use a mass attenuation coefficient, $(\mu/\rho)_{Pb}$, of $0.058\ \mathrm{cm^2\ g^{-1}}$ for these rays in lead (with a density, ρ, of $11.3\ \mathrm{g\ cm^{-3}}$). **1.0 pt**

Given the availability and durability of $1\ \mathrm{mm}$ thick stainless-steel sheets in the facility, your team is considering stainless steel as an alternative to the proposed lead shield. The density of stainless steel is $8.03\ \mathrm{g\ cm^{-3}}$. **Table 1** provides the elemental mass attenuation coefficients, $(\mu/\rho)_i$, and weight fractions (w_i) of each element in stainless steel.

Table 1. *Elemental composition of stainless steel, showing the weight fraction (Wt%) and mass attenuation coefficient (MAC) for each element*

Elements	Wt%	MAC
C	0.0008	0.0568
Mn	0.02	0.0521
P	0.00045	0.0551
S	0.0003	0.0568
Si	0.01	0.0567
Cr	0.19	0.0528
Ni	0.095	0.0548
Fe	0.68345	0.0534

2.4 **(a)** Calculate the effective mass attenuation coefficient, $(\mu/\rho)_{ss}$, of stainless steel using the equation: **0.5 pt**

$$\left(\frac{\mu}{\rho}\right)_{ss} = \sum_i w_i \times \left(\frac{\mu}{\rho}\right)_i \qquad (7)$$

where w_i is the weight fraction and $(\mu/\rho)_i$ is the mass attenuation coefficient of the i-th element.

(b) Calculate how many $1\ \mathrm{mm}$ thick stainless-steel sheets are needed to achieve the desired shielding effect that keeps the egg hatch rate close to 2%. Round up fractional results to use full $1\ \mathrm{mm}$ sheets. **0.5 pt**

Part 3. Competitiveness and Dose in SIT (2.0 pts)

In the ongoing battle against mosquito-borne diseases, scientists have noticed an intriguing trend. As the irradiation dose increases, the mating competitiveness of the sterile male mosquitoes initially experiences a gradual decline. Competitiveness is defined as their ability to successfully compete with wild males for mating with wild females. However, beyond a certain dose threshold, this decline becomes more pronounced, leading to a sharp drop in competitiveness. This pattern is depicted in **Figure 2**, underscoring the critical role of precise dose management in optimizing the SIT for mosquito control. For the following problems, assume that the irradiation times are fixed.

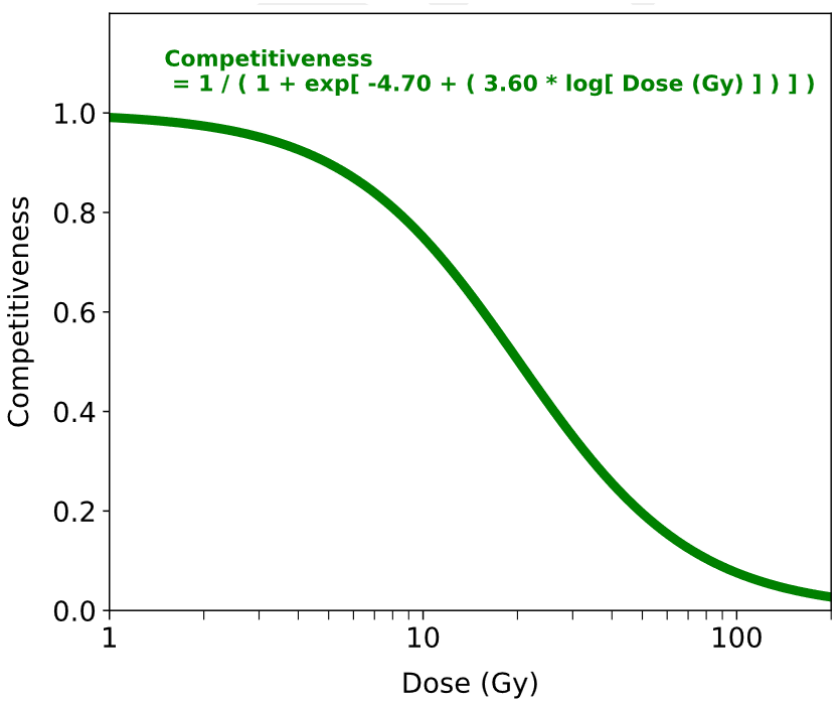

***Figure 2**: Relationship between irradiation dose and mating competitiveness*

3.1 Given the equation for male competitiveness (C) as a function of dose: **1.0 pt**

$$C = \frac{1}{1+exp(-4.7+[3.6\times\log(Dose)]} \qquad (8)$$

If an irradiation dose of 40 Gy is administered, calculate the resulting competitiveness.

3.2 Conversely, if specific competitiveness level of 0.5 is desired, calculate the irradiation dose required to achieve this level. **1.0 pt**

Q3 PARTICLES FOR DESTROYING CANCER (10 pts)

Important Notes:

1. Do NOT round off figures during calculations. Round off final answers to three significant figures.
2. Express final answers in SI units.
3. Use scientific notations to express very small or very large values.
4. Refer to Sheet G2 for useful equations, constants & conversion factors.

Part 1. Proton Stopping Power, Range, and Dose (5.0 pts)

High-energy particles entering a medium interact with matter and lose energy to varying extents depending on the type of particle, energy, charge, and mechanism of interaction. These interactions gradually slow and eventually stop the particles. The ability of a material to stop a charged particle is called the linear stopping power, S.

1.1 Given the expressions below for S and the linear energy transfer LET, what is the fundamental difference between S and LET? **0.4 pt**

$S = -dE/dx$ (1)

$LET = dE/dx$ (2)

Note that E is the particle energy and the x is the distance travelled by the particle.

The stopping powers of photons and charged particles vary due to their different interactions with matter. Understanding the mechanisms by which radiation interacts with matter is essential for understanding how dose is deposited in a target material, which has significant implications in medical physics and radiation therapy.

Current clinical practice for use of external beam radiation in cancer treatment includes photon and electron therapy. However, photon therapy delivers excess dose to healthy tissues both proximal and distal to the treatment site. In contrast, electron therapy results in a minimal dose beyond the treatment site, but excessive multiple Coulomb scattering restricts its use to areas within 6 cm of the target surface. Protons, being mass particles, slow down faster than photons and deposit more energy as they slow down, culminating in a dose peak known as the Bragg peak.

The graph below shows the different energy deposition of photons and charged particles based on their stopping powers as they traverse through matter

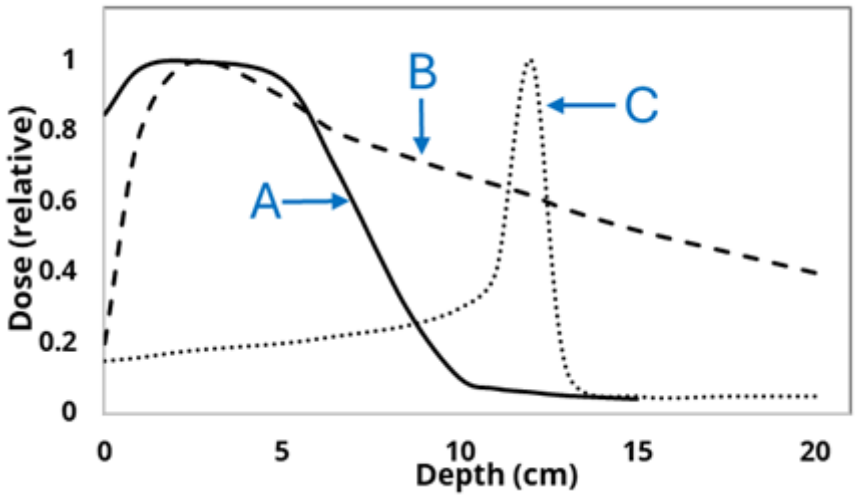

***Figure 1**. Depth-dose distribution of different radiation treatments in water*

1.2	Identify the corresponding curve in **Figure 1** that represents the dose-depth distribution for electrons $(20\ \mathrm{MeV})$, photons $(18\ \ \mathrm{MeV})$, and protons $(130\ \mathrm{MeV})$ in matter (e.g., water).	**0.6 pt**

Understanding stopping power can generate insight on a new way to fight cancer. The mass stopping power is the linear stopping power divided by the material density ρ, S/ρ (in $\mathrm{J}\cdot\mathrm{m}^2\cdot\mathrm{kg}^{-1}$) and can be measured or calculated for a given material. The calculation of mass stopping power and range R of protons plays an important role in proton radiation therapy.

The Bethe-Bloch formula describes the energy loss of charged particles (such as protons) as they traverse through a material. Specifically, it describes the mass stopping power of a material for a charged particle. The Bethe-Bloch formula for the mass stopping power of protons at 1-200 MeV energy range is given in Equation 3. The combination of the $1/{\beta_v}^2$ and the logarithmic terms explain the origin of the Bragg peak.

$$\frac{S}{\rho} = -\frac{dE}{\rho dx} = \frac{5.08\times10^{-31}z^2n}{\rho{\beta_v}^2}\left[F\left(\beta_v\right) - \ln\ \left(I\right)\right] \tag{3}$$

$$F\left(\beta_v\right) = \ln\frac{1.02\times10^6{\beta_v}^2}{1-{\beta_v}^2} - {\beta_v}^2 \tag{4}$$

$$n = \frac{N_A Z\rho}{A} \tag{5}$$

where β_v is the velocity of the incident particle v relative to the speed of light c; I is the excitation energy of a target material ($I_{water} = 74.6$ eV); z is the charge of the incident particle ($z_{proton} = +1$); and n is the number of electrons in a material per unit volume (in m^3), calculated in terms of Avogadro's number (N_A), material density ρ_{water}, the material atomic number ($Z_{water} = 10$), and the material mass number ($A_{water} = 18$).

1.3	Simplify equation (3) for the mass stopping power S/ρ of water for protons.	**0.5 pt**

1.4	Using the derived equation from 1.3, calculate the following at kinetic energies E_K of 1, 10, and 100 MeV, assuming that protons are moving at relativistic speeds:	
	(a) proton velocity	**0.75 pt**
	(b) mass stopping power of water	**0.75 pt**
	(c) Bragg-Kleeman range R (in cm) in water as a function of the incident energy E_K. Note: $R = N_R \times {E_K}^{\beta_e}$ where $N_R = 0.0023$ g cm^{-2} MeV^{-1}, $\beta_e = 1.75$	**0.75 pt**
	(d) What general relationship can you derive between proton energy, mass stopping power, and range? Create a diagram by plotting the obtained values.	**0.5 pt**

The physical dose D at a point in a radiation field is the average energy absorbed E_{abs} per unit target mass m. The equation (6) relating dose to mass stopping power is the starting point of most proton beam line design programs. The proton fluence Φ, which is defined as the number of protons N_P per unit area A, can be used to compute the dose as shown below:

$$D \equiv \frac{E_{abs}}{m} = \frac{(dE/dx)\times dx \times N_P}{\rho \times A \times dx} = \Phi \frac{dE}{\rho dx} \quad (6)$$

The distance protons travel past a marker position is the residual range R_{res}. With simplifying approximations that neglect the influence of nuclear reactions and Coulomb scattering, it is assumed that (1) the proton energy loss and the energy absorbed by the target material are proportional, and (2) based on the continuous slowing down approximation (CSDA), the rate of energy loss is continuous with distance (range) as the proton decelerates.

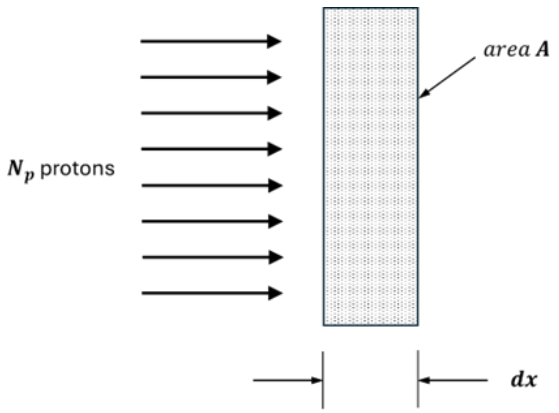

Figure 2. *Proton beams impinging on a target.*

Table 1. *Proton Stopping Power and Range (in water, liquid)*

Kinetic Energy (MeV)	S, total (MeV cm^2 g^{-1})	Range, CSDA (g cm^{-2})
5.000E+01	1.245E+01	2.227E+00
5.500E+01	1.154E+01	2.644E+00
6.000E+01	1.078E+01	3.093E+00
6.500E+01	1.013E+01	3.572E+00
7.000E+01	9.559E+00	4.080E+00
7.500E+01	9.063E+00	4.618E+00
8.000E+01	8.625E+00	5.184E+00
8.500E+01	8.236E+00	5.777E+00
9.000E+01	7.888E+00	6.398E+00
9.500E+01	7.573E+00	7.045E+00
1.000E+02	7.289E+00	7.718E+00

1.5 A proton beam of $100\ \mathrm{MeV}$ and fluence of $1 \times 10^9\ \mathrm{protons\ cm^{-2}}$ is incident on water that is $5.491\ \mathrm{cm}$ thick in the beam direction. Ignoring nuclear reactions, what is the average dose to the water $\left(\rho_{water} = 1\ \mathrm{g\ cm^{-3}}\right)$? For $100\ \mathrm{MeV}$ protons in water, $R_{CSDA} = 7.718\ \mathrm{g\ cm^{-2}}$. Note that $R_{res} = R_{CSDA} - thickness$ and that R_{CSDA} has a corresponding proton kinetic energy, which can be found in **Table 1**. Use this information to solve for proton absorbed dose. **0.75 pt**

Part 2. Proton Therapy (5.0 pts)

Protons lose energy and deposit most of their dose over a short distance (depth) in a pattern determined by the beam energy and the target medium. By adjusting the energy, physicians can precisely control when and where the protons release most of the energy. Combining proton beams of different energies creates a Spread-Out Bragg Peak (SOBP) with a plateau-like dose that can damage a tumor across its full depth (**Figure 3**).

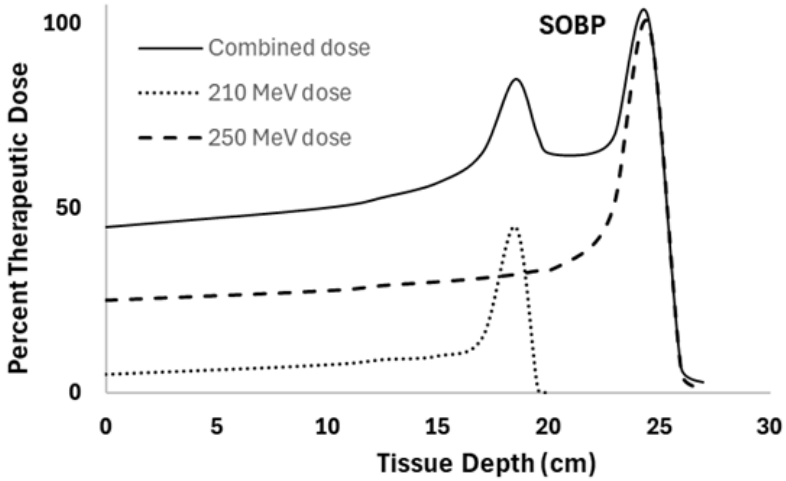

***Figure 3**. Percent therapeutic dose deposited in tissue by proton beams. The combined dose is offset for illustration only.*

2.1	**(a)** Using an arrow, indicate the specific region in the graph where the highest LET occurs.	**0.3 pt**
	(b) (True or False) As the proton slows down, the rate of energy loss is proportional to the square of the particle charge and the square of velocity.	**0.3 pt**
	(c) (True or False) In proton therapy, the exit dose is less of a concern compared to photon therapy.	**0.3 pt**
	(d) To treat a tumor at a depth of $18\ \mathrm{cm}$ to $25\ \mathrm{cm}$, which proton energy would a physician likely add to the SOBP in **Figure 3**? A. $260\ \mathrm{MeV}$ B. $210\ \mathrm{MeV}$ C. $200\ \mathrm{MeV}$ D. $235\ \mathrm{MeV}$	**0.3 pt**

The amount of radiation (dose) to cause a biological effect is measured in terms of relative biological effectiveness (RBE), which compares the dose required to produce the same biological effect between a radiation beam (often charged particles) and a reference radiation (photon), usually defined as $250\ \mathrm{keV}$ X-rays or ^{60}Co γ-rays. For the same radiation dose, high LET radiation (e.g., alpha particles, protons) will cause more damage than low LET radiation (e.g., X-rays). Radiation doses are considered isoeffective if the dose of a treatment carried out under reference conditions will produce the same clinical effects on the target. The current clinical guideline for proton therapy uses a constant RBE value $(RBE = 1.1)$ relative to high-energy photons, which accounts for the higher biological effectiveness of protons (p) over photons (x).

Several models developed for predicting RBE all rely on the linear quadratic (LQ) model of cell survival, which is one of the key tools in radiation biology and physics.

$$S\Big(D\Big) = e^{(-\alpha D+\beta D^2)} \tag{7}$$

The LQ model has been used extensively to analyze and predict responses to ionizing radiation both in vitro and in vivo. In this model, $S(D)$ is the fraction of cells that survive a delivered dose (D). The main parameters of this model, α and β, indicate how resistant a cell type is to radiation damage: cells with a higher (α/β) (~ 10) are less sensitive to a high dose per fraction.

In the LQ model (equation 7), if a proton absorbed dose D_{p} and a photon dose D_{x} are isoeffective, the relationship is expressed as:

$$\alpha_p D_p + \beta_p D_p^2 = \alpha_x D_x + \beta_x D_x^2 \tag{8}$$

$$\beta_x D_x^2 + \alpha_x D_x - \alpha_p D_p - \beta_p D_p^2 = 0 \tag{9}$$

2.2 Derive an LQ model based on equation (9) to solve for positive value of RBE *or* $\left(\frac{D_x}{D_p}\right)$. Express applicable terms as $\left(\frac{\alpha}{\beta}\right)_x$, RBE_{max}, or RBE_{min} using the following expressions: $RBE_{max} = \frac{\alpha_p}{\alpha_x}$ and $RBE_{min} = \sqrt{\frac{\beta_p}{\beta_x}}$. **1.0 pt**

RBE depends on the type of particle, the dose-averaged linear energy transfer (LET_{d}), the cell or tissue type defined by $(\alpha/\beta)_x$, and the dose per fraction (D_{p}).

Table 2. *Fit parameters*

Parameter	Values
$RBE_{max} = p_0 + \frac{p_1}{(\alpha/\beta)_x} LET_d$	$p_0 = 0.999064$
	$p_1 = 0.35605 \text{ Gy } (\text{keV } \mu\text{m})^{-1}$
	$p_2 = 1.1012$
$RBE_{min} = p_2 + p_3 \sqrt{\left(\frac{\alpha}{\beta}\right)_x} LET_d$	$p_3 = 0.0038703 \text{ Gy}^{-\frac{1}{2}} (\text{keV } \mu\text{m})^{-1}$

Table 3. *Cell lines with corresponding* $\left(\alpha/\beta\right)_x$

Cell line	$\left(\alpha/\beta\right)_x$
HaCat	15.0
SKMel	3.0

2.3 **(a)** Using the LQ model derived in 2.2 and the fit parameters in **Table 2**, calculate the RBE for the two cell lines in **Table 3** as a function of proton dose $(D_p = 2 \text{ Gy})$ and LET_d $\left(1.9\ ,\ 2.5\ ,\ 4.5 \text{ keV } \mu\text{m}^{-1}\right)$. **1.2 pt**

(b) How does the model predict the RBE of protons in terms of the LET_d and cell radiosensitivity $\left(\alpha/\beta\right)_x$? **0.4 pt**

(c) Calculate the isoeffective photon dose D_x from the obtained *RBE* values fat $D_P = 2 \text{ Gy}$ **1.2 pt**

Q4 MASS ABUNDANCE OF ISOTOPES (10 pts)

The equivalence between mass and energy, as suggested by Einstein's Special Theory of Relativity, has a profound effect on the field of nuclear science and technology. Because the binding energy of the nucleons inside an atomic nucleus is very high (typically in the range of MeV as compared to the electron binding energy to the nucleus which is in eV or keV range), the energy that appears as the nuclear binding energy is a considerable fraction of the energy which appears as the nuclear mass. Therefore, the mass of a nucleus ${}^{A}_{Z}X_{N}$ is not simply equal to $(Zm_p + Nm_n)$ where m_p and m_n are proton mass and neutron mass, respectively. This fact makes the accurate theoretical calculation of the atomic or nuclear masses impossible, particularly for nuclides with higher mass numbers. Hence, the precise finding of atomic masses relies on experimental methods. The typical experimental apparatus used to determine the nuclear masses and relative abundances of isotopes in samples is either a mass spectrograph or mass spectrometer, which are based on the same principle of operation. A simplified schematic diagram, of cross-sectional view, showing the main components of a mass spectrograph is shown in **Figure 1**.

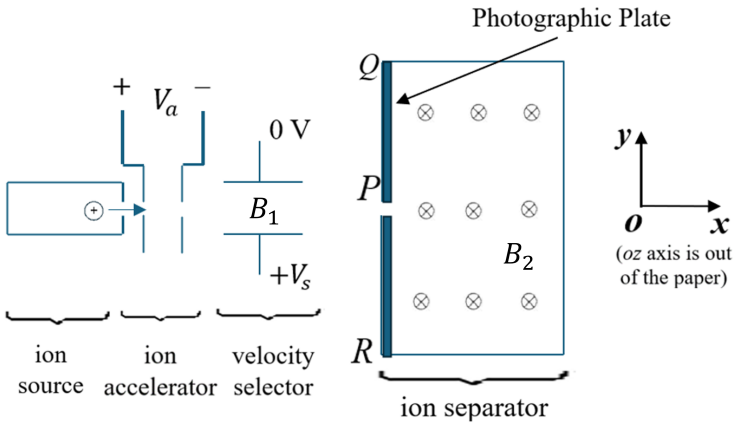

***Figure 1**. Schematic diagram of mass spectrograph*

A beam of ionized atoms is generated in the ion source. The generated ions normally have a broad distribution of velocity. These ions are accelerated to a higher energy by passing them through a potential difference in the ion accelerator. Inside the velocity selector, an electric field and a uniform magnetic field (B_1) are applied so only ions with a definite velocity enter the ion separator. Ions then make their way into the ion separator in which they travel under the influence of a second uniform magnetic field (B_2). When ions hit the photographic plate, an image is produced at the point of incidence. By measuring the position of the image, the mass of the ion can be inferred. The regions through which the ions are traveling are maintained at high vacuum conditions.

Part 1. Basic Operation of a Mass Spectrograph (2.1 pts)

For the calibration of a particular mass spectrograph, a beam of ^{12}C isotope for which the mass is taken to be exactly $12.000000\ \mathrm{u}$ on the atomic mass scale is used. Assume that singly ionized carbon ions are generated in the ion source.

1.1	If $V_a = 1.20\ \mathrm{kV}$, calculate the increase in kinetic energy (in joules) of an ion after passing through the ion accelerator.	**0.3 pt**

1.2	For an ion that exits the ion source with almost zero speed, what is the velocity at the end of the ion accelerator? You may disregard the mass of an electron compared to the atomic mass of ^{12}C.	**0.4 pt**

Electric and magnetic fields in the velocity selector are adjusted so that ions traveling along a straight line and entering the ion separator would have a velocity of $140.00\ \mathrm{km/s}$.

1.3	**(a)** What should be the direction of the magnetic field B_1 (ox, oy, oz, $-ox$, $-oy$ or $-oz$)?	**0.3 pt**
	(b) If $V_s = 400.0\ \mathrm{V}$ and the distance between the plates of the velocity selector is $2.0\ \mathrm{cm}$, calculate the magnetic flux density (in tesla) of B_1.	**0.7 pt**
	(c) If the ^{12}C is doubly ionized, what is their velocity after passing through the velocity selector?	**0.4 pt**

Part 2. Motion of Ions Inside the Ion Separator (2.9 pts)

In **Figure 1**, it is given that $PQ = PR = 148\ \mathrm{cm}$, and the magnetic flux density of B_2 is $0.030\ \mathrm{T}$. Consider an ion entering the ion separator through the hole at P with the same velocity as given in part 1.3 , i.e. $140.0\ \mathrm{km/s}$.

2.1	For a singly ionized atom, find the distance (in cm) to the point where the ion hits the photographic plate from point P.	**0.7 pt**

2.2	The resolution of the distance measurements for this mass spectrograph is $1\ \mathrm{mm}$. What is the minimum atomic mass difference $(\Delta m)_{min}$ (in u) between two elements or isotopes that can be resolved using this spectrograph? Express your answer in units of u. Assume that the uncertainties related to electric and magnetic fields are negligible.	**1.0 pt**

2.3	Assume that the ion beam produced in the ion source contains **both** singly and doubly ionized ^{12}C atoms. Draw the paths of these ions, identifying each separately, in the given Figure in the answer sheet. For each ion, indicate the point of incidence as accurately as possible according to the scale provided in the answer sheet.	**1.2 pts**

Part 3: Mass Spectrometers (1.6 pts)

The range of atomic (or nuclear) masses that can be analyzed with a mass spectrograph, for a particular value of the magnetic field strength of B_2, is not very large. In addition, it is not possible to quantitatively estimate the abundance of different elements or isotopes of a given element from the information extracted from the photographic plate. These drawbacks present in mass spectrographs can be eliminated by replacing the photographic plate in the ion separator with an electric current measuring detector installed at an exit slit at a particular point on the focal plane of the apparatus as shown in **Figure 2**. Note that all other parts are the same as given in Part 1. This set up is known as the *mass spectrometer*.

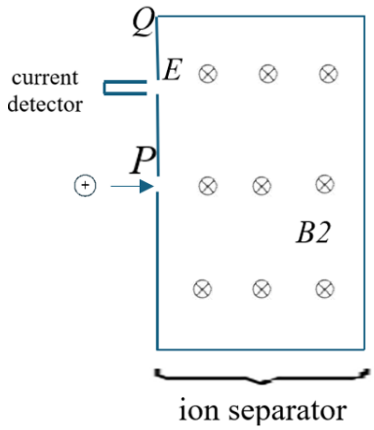

***Figure 2**. Ion separator for a mass spectrometer*

Unlike in the mass spectrograph, the flux density of the magnetic field B_2 is not constant here. The entering ion beam into the ion separator is focused onto a very narrow exit slit (*E*) by changing B_2. If the entering ion beam is a mixture of different isotopes, each isotope can be focused onto the slit, one at a time. The current at the exit slit is measured using a current detector installed at the slit.

3.1	Consider a mass spectrometer for which the electric and magnetic field settings of the velocity selector are the same as given in part 1.3 (b). If the beam exit slit E is located at a distance $80.0\ \mathrm{cm}$ from P and the flux density B_2 can be set at any value between 20.0 and $320.0\ \mathrm{mT}$, estimate the minimum and maximum atomic masses (in u) that can be analyzed using this spectrometer. Assume that all ions are singly charged.	**1.0 pt**

3.2	When an ion beam produced from naturally occurring carbon is analyzed using the spectrometer described above, the currents measured for two stable isotopes ^{12}C and ^{13}C, when they are separately focused on the exit slit, are $16.5\ \mathrm{mA}$ and $185\ \mu A$, respectively. Find the ratio $\frac{Abundance\ of\ ^{12}C}{Abundance\ of\ ^{13}C}$. Assume that all the ions are singly charged.	**0.6 pt**

Part 4: Determining the Age of Rock samples (3.4 pts)

In geology, determining the age of rock samples found in the Earth's environment is essential in understanding and validating the different hypotheses related to the formation of the earth, and the formation of the planetary system in general. Very often, the determination of the age of a sample requires the measurement of the ratios between the abundance of different elements or different isotopes of the same element. As illustrated in Part 3, these ratios can be found, with reasonable accuracy, using mass spectrometry techniques.

Consider a sample of rock that contains a radioactive parent element P, stable daughter element D which is created from the decay of P, and another element D_s, which is a stable isotope of D. The number of nuclei of each of these elements, as determined at the present time is indicated by t_1, are $N_P(t_1)$, $N_D(t_1)$ and $N_{D_s}(t_1)$, respectively. Also assume that the rock sample was formed at a time t_0 at which time the corresponding number of nuclei are $N_P(t_0)$, $N_D(t_0)$ and $N_{D_s}(t_0)$.

4.1 **(a)** Write down a relationship between $N_P(t_0)$, $N_D(t_0)$ and $N_D(t_1)$. **0.4 pt**

(b) If the decay constant for the process $P \rightarrow D$ is λ, and the ratio $\frac{N_D(t_1)}{N_{D_s}(t_1)}$ is expressed as $\frac{N_D(t_1)}{N_{D_s}(t_1)} = G\frac{N_P(t_1)}{N_{D_s}(t_1)} + \frac{N_D(t_0)}{N_{D_s}(t_0)}$, find the coefficient G in terms of λ, t_0 and t_1. **0.7 pt**

If many rock samples had been formed from the same source (e.g. through the condensation of smaller parts from the same gas cloud), then it can be hypothesized that all these rock samples would have had the same $\frac{N_D(t_0)}{N_{D_s}(t_0)}$ ratio. Under this condition the relationship between $\frac{N_P(t_1)}{N_{D_s}(t_1)}$ and $\frac{N_D(t_1)}{N_{D_s}(t_1)}$, measured for different rock samples, becomes a linear relationship.

Radioactive decay of ^{87}Rb (half-life $t_{1/2} = 4.8 \times 10^{10}$ y produces stable ^{87}Sr isotope. Four rock samples have been analyzed using a mass spectrometer, similar to the one described in Part 3. The measured current values for ^{87}Rb, ^{87}Sr and ^{86}Sr (a stable isotope of ^{87}Sr are given in **Table 1** You may identify ^{87}Rb, ^{87}Sr and ^{86}Sr as the elements P, $D$and D_s, respectively.

Table 1. Measured currents for the rock samples

Rock Sample	Measured Current (mA)		
	^{87}Rb	^{87}Sr	^{86}Sr
1	4.8	1.65	1.91
2	0.55	0.81	1.10
3	0.8	0.37	0.46
4	7.4	1.72	1.80

4.2 **(a)** Based on the current measurements given in **Table 1**, complete the table in the answer sheet. Indicate your answers in 3 significant figures. **0.8 pt**

(b) Using the data from Part 4.2 (a), plot an appropriate graph that would allow the determination of the age of the rock samples. Clearly indicate the variables you selected for each axis of the graph. **0.6 pt**

(c) Find the gradient of the graph you plotted in part 4.2 (b). Indicate your answer in 3 significant figures. **0.3 pt**

(d) Determine the age of the rock samples in years. **0.6 pt**

Q5 ADVANTAGES OF USING LOW ENRICHED URANIUM IN NUCLEAR REACTORS (10 pts)

Light water reactors (LWRs) used for power generation are fueled with low enriched uranium (LEU) having 3% to 4% enrichment (weight % of ^{235}U), while experimental/research reactors use up to slightly less than 20% enriched uranium. Fission reactions are induced mainly in fissile ^{235}U to produce energy and neutrons to keep the reactor critical (steady-state). ^{235}U is the only naturally-occurring fissile nuclide and if all the energy is to be generated solely by fissioning of this material, the limited resources for practical fission fuel will be depleted faster. The heavier isotope of uranium, ^{238}U, also contributes to energy production through fast fission and conversion to the fissile nuclide ^{239}Pu.

In this problem, it will be studied how fissioning and conversion of ^{238}U saves ^{235}U and natural uranium resources, while also reducing the production of ^{236}U that is a long-lived radioactive waste with higher activity per unit mass. Furthermore, the problem first addresses the nuclear non-proliferation aspect of the enrichment process and concludes with the criticality safety implications of handling high enriched uranium (HEU).

In this problem, **answers should be reported to four (4) significant digits**. In addition, **no truncation** should occur in any intermediate calculations, i.e. if a result from a subproblem is used in subsequent parts, calculations should proceed without rounding off.

Part 1. Uranium Enrichment (3.5 pts)

Uranium enrichment is achieved by separating ^{235}U and ^{238}U in natural uranium, i.e. uranium having natural isotopic abundance. A typical procedure inserts uranium hexafluoride (UF_6) into a device that separates the isotopes. The device will have a feeder where mass M_F of natural uranium with enrichment x_F (in weight fraction) is inserted. The device will then produce enriched uranium with mass M_P and enrichment x_P, and depleted uranium, also called tails, with mass M_T and enrichment x_T. Intuitively, $x_P > x_F > x_T$. The material flow is depicted in **Figure 1**.

Figure 1. *Material flow in an enrichment plant.*

1.1	**(a)** Consider an ideal enrichment plant. Write the expressions for conservation of mass of uranium and just ^{235}U content.	**0.1 pt**
	(b) Derive the expression relating M_F and M_P.	**0.3 pt**

The amount of ^{235}U in the product can be calculated as follows: $M_{235} = x_P M_P$.

1.2	Assuming that $x_T = 0.002$, with $x_F = 0.72\%$ as the natural abundance of ^{235}U, express the feed mass M_F in terms of the ^{235}U mass (M_{235}) in the product for the following enrichment levels:	
	(a) 3% for a nuclear power plant fuel	**0.5 pt**
	(b) 19.7% for a research reactor fuel such as TRIGA	**0.5 pt**

The separative work unit (SWU) measures the work an enrichment plant performs to separate ^{235}U and ^{238}U isotopes. Calculating SWU is crucial for the economics of enrichment plants and nuclear fuel costs. SWU is calculated using the expression below:

$$SWU = M_P[V(x_P) - V(x_T)] - M_F[V(x_F) - V(x_T)] \quad (1)$$

where the function $V(x)$ is known as the value function, and is given by:

$$V(x) = (1 - 2x) \ \ln\left(\frac{1-x}{x}\right) \quad (2)$$

where x is the enrichment in weight fraction.

1.3	**(a)** With the same value of x_T and x_F given in part 1.2, and with $M_F = 1000$ kg, calculate for the SWU of the following values of uranium enrichment: 1.0%, 3.0%, 10.0%, 20.0%, 50.0%, and 90.0%.	**1.2 pt**
	(b) Plot x_p as a function of SWU. Indicate y-axis values. Uranium that can be weaponized requires an enrichment that is higher than 90%.	**0.6 pt**
	(c) What can be said about the separative work required to enrich uranium from a low value up to 20% compared to going higher than this threshold for commercial and civilian reactor fuel? Note: The enrichment limit is a measure to prevent the proliferation of nuclear weapons.	**0.3 pt**

Part 2. Energy from ^{235}U and ^{238}U (4.0 pts)

In LWRs, slow neutrons interacting with fissile nuclei are more likely to undergo radiative capture and fission due to their significantly higher interaction cross section (σ) compared to other interactions. ^{235}U that interacts via radiative capture is converted into radioactive ^{236}U. Radiative capture cross section (σ_γ) can be obtained from the absorption cross section (σ_a) and the fission cross section (σ_f) through the following relation: $\sigma_\gamma = \sigma_a - \sigma_f$.

Table 1. *Thermal neutron cross section data for fissile nuclides*

Fissile Nuclei	σ_a(barns)	σ_f(barns)
^{233}U	578.8	531.1
^{235}U	680.8	582.2
^{239}Pu	1011.3	742.5
^{241}Pu	1377.0	1009.0

Consider a hypothetical region in a country with a 5×10^{10} kWh annual electricity requirement. Assume only nuclear power plants supply this electricity demand and that the plants operate with an efficiency of 30%. Of the 200 MeV total energy released in a fission process, 5% is carried away by neutrinos and is not absorbed within the nuclear reactor. By approximating the atomic mass values using the mass numbers of the nuclides, the following are some important metrics if we consider only fissions occuring with ^{235}U and 365 days a year.

Table 2. *Annual Parameters for Nuclear Power Generation in Hypothetical Region (only ^{235}U as fuel)*

^{235}U probability for fission upon neutron absorption	0.8552
^{239}Pu probability for fission upon neutron absorption	0.7342
Total Net power	5708 MW
Mass of ^{235}U fissioned	7691 kg
Total Mass of ^{235}U consumed	8994 kg
Total mass of natural uranium used	1.631×10^6 kg
Total mass of ^{236}U produced	1308 kg

Neutrons in a reactor are also absorbed by ^{238}U along with their absorption in ^{235}U. In thermal reactors, most of the neutrons absorbed by ^{238}U convert it into ^{239}Pu, through the following reaction.

$$n + {}^{238}_{92}U \longrightarrow {}^{239}_{92}U + \gamma \xrightarrow{\beta^-} {}^{239}_{93}Np \xrightarrow{\beta^-} {}^{239}_{94}Pu \tag{3}$$

As previously mentioned, most modern reactors use LEU fuel, primarily composed of ^{238}U. Consequently, the conversion of ^{238}U to ^{239}Pu occurs routinely during normal reactor operation. In a typical thermal reactor fueled with LEU, 20% of the fission neutrons are used to convert ^{238}U to ^{239}Pu due to resonance absorptions. In addition, out of the overall fissions, about 35% occur in plutonium and 5% in ^{238}U during the fuel's residence time in the reactor core.

2.1	Assuming that 2.6 neutrons are produced per fission, how many kilograms of plutonium are produced after one year in all the reactors that are used to generate 5×10^{10} kWh of electricity?	**0.6 pt**

2.2	Assume that nuclear power plants use 3.5 wt.% enriched uranium, and the enrichment plants are operating with $x_T = 0.002$, and $x_F = 0.0072$.	
	(a) Calculate the amount in kilograms of ^{235}U fissioned.	**0.6 pt**
	(b) Calculate the amount in kilograms of ^{235}U consumed.	**0.6 pt**
	(c) Calculate the amount of natural uranium (in kilograms) needed to produce the required fissile mass as calculated in 2.2b.	**0.8 pt**
	(d) Comparing to the values in **Table 2**, provide the amount of natural uranium saved in kilograms due to contribution of ^{239}Pu and fast fission of ^{238}U.	**0.2 pt**

2.3	What are the masses of ^{236}U, and ^{240}Pu present in all the reactors after one year?	**1.2 pt**

Part 3. Criticality Accident (2.5 pts)

In addition to preventing the proliferation of weapons-useable material, use of LEU in commercial power reactors also ensures safety in the design by reducing the likelihood of a criticality accident. Criticality accident is the release of energy from an accidental production of a self-sustaining or divergent fission chain reaction. An example of a criticality accident that occurred with HEU is explored here.

On April 5, 1968, a criticality accident occurred at the Russian Federal Nuclear Center (VNIITF) located in the southern Ural Mountains between the cities of Ekaterinburg and Chelyabinsk. Criticality experiments have been done at VNIITF since 1957 in support of developing reactors for studying radiation tolerance of materials. One of the devices operated is the FKBN vertical lift assembly machine, with an assembly of various critical configurations of spherical metal shell with a large internal cavity surrounded by a thick reflector. This configuration allowed static and pulsed mode operations enabling research on kinetic behavior of reactor systems.

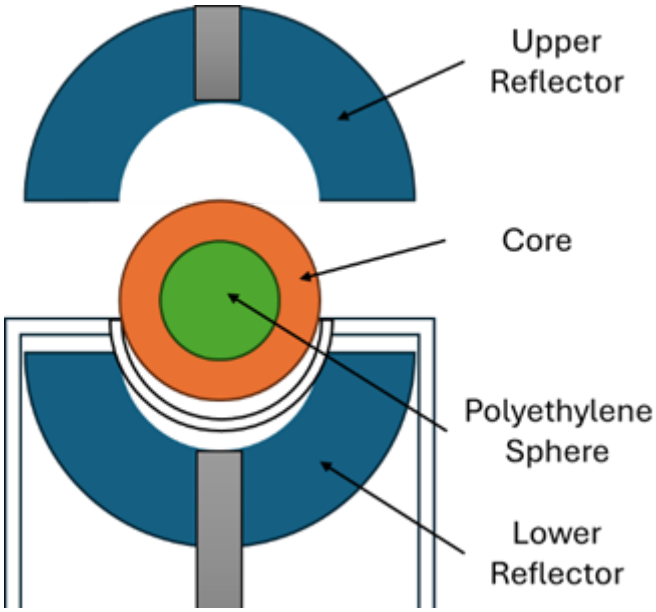

Figure 2. *Approximate accident configuration of the FKBN assembly.*

The configuration of the FBKN assembly at the time of the accident is shown in **Figure 2**. It consists of a 90% enriched (wt.%) uranium spherical metal shell core having a mass of $47.7\ \mathrm{kg}$, with an internal cavity containing a solid polyethylene sphere. The surrounding spherical reflector is made of natural uranium metal with an inside radius of $91.5\ \mathrm{mm}$ and outside radius of $200\ \mathrm{mm}$ that is split into upper and lower hemispheres. The accident occurred as the upper half of the reflector

was being lowered onto the core leading to two personnel fatalities due to the radiation burst. The excursion resulted in an estimated yield of 6×10^{16} fissions. It was identified that the incorrect position of the lower reflector was the primary cause of the accident.

3.1	What is the energy released in the accident in units of kilograms of high explosive material equivalent (1 J = 2.390×10^{-7} kg of TNT)? Assume 200 MeV is released per fission event.	**0.5 pt**

One of the victims of the criticality accident is the senior criticality safety specialist. He was located 1.7 m away from the device with an estimated accumulated neutron plus gamma dose in the range of $5 - 10$ Sv. **Table 3** shows the parameters relevant to the external dose calculation for $^{235}U(n, f)$ reaction.

Table 3. *Parameters for $^{235}U(n, f)$ reaction relevant for external dose calculation*

Parameter	Value
Average prompt gamma yield / fission	8.58
Average Gamma Ray Energy (MeV)	0.85
Photon Fluence – Dose Coefficient (pSv · cm^2) at 0.85 MeV	3.92
Average neutron yield / fission (2.0 MeV incident neutron)2.80	2.80
Prompt Neutron Average Energy (MeV)	2.0
Neutron Fluence – Dose Coefficient (pSv · cm^2) at 2.0 MeV	407

The unattenuated effective dose (E) for a point source can be approximated by:

$$E = \frac{C_D S}{4\pi r^2} \tag{4}$$

where C_D is the appropriate fluence-dose conversion coefficient, S is the source strength (particle/s), and r is the distance from the source.

3.2	Estimate the effective dose in Sv from photons for the senior specialist using point source approximation and neglecting the shielding and scattering effects of the natural uranium reflector.	**1.0 pt**

3.3	Estimate the effective dose from neutrons, and the total effective dose using point source approximation. Which ionizing radiation contributed most to the dose?	**1.0 pt**