Q1 APPLICATION OF FUSION REACTION (10 pts)

Part 1. Energy from fusion reaction (2.0 pts)

1.1 (a) Given the following D-D and D-T nuclear reactions:

D-D: $^{2}_{1}\mathrm{H} + ^{2}_{1}\mathrm{H} \rightarrow ^{3}_{2}\mathrm{He} + ^{1}_{0}\mathrm{n}$

D-T: $^{2}_{1}\mathrm{H} + ^{3}_{1}\mathrm{H} \rightarrow ^{4}_{2}\mathrm{He} + ^{1}_{0}\mathrm{n}$

The Q-value for the D-D reaction is:

$$Q = [(m_A + m_B) - (m_C + m_D)]c^2$$

$$Q = [(2.014102\ \mathrm{u} + 2.014102\ \mathrm{u}) - (3.016029\ \mathrm{u} + 1.008665\ \mathrm{u})](931.5\ \mathrm{MeV/u})$$

$$Q = (0.003510\ \mathrm{u})(931.5\ \mathrm{MeV/u}) = \mathbf{3.270\ MeV}$$

The Q-value for the D-T reaction is:

$$Q = [(m_A + m_B) - (m_C + m_D)]c^2$$

$$Q = [(2.014102\ \mathrm{u} + 3.016049\ \mathrm{u}) - (4.002603\ \mathrm{u} + 1.008665\ \mathrm{u})](931.5\ \mathrm{MeV/u})$$

$$Q = (0.018883\ \mathrm{u})(931.5\ \mathrm{MeV/u}) = \mathbf{17.59\ MeV}$$

1.1 (a)	D-D:	3.270 MeV	0.1 pt
	D-T:	17.59 MeV	0.1 pt

1.1 (b) The Q-value is equal to the sum of the kinetic energy of the neutron (E_n) and the other product (E_x):

$$Q = E_n + E_x = \frac{1}{2}m_n v_n^2 + \frac{1}{2}m_x v_x^2$$

Since momentum is conserved, and it is initially zero, then:

$m_n v_n = m_x v_x$, from which we get:

$$v_x = \frac{m_n v_n}{m_x}$$

Substituting the expression for v_x into the first equation:

$$Q = E_n + E_x = \frac{1}{2}m_n v_n^2 + \frac{1}{2}m_x\left(\frac{m_n v_n}{m_x}\right)^2$$

$$Q = \frac{1}{2}m_n v_n^2\left(1 + \frac{m_n}{m_x}\right) = E_n\left(\frac{m_x + m_n}{m_x}\right)$$

Solving for E_n, we obtain:

$$\boldsymbol{E_n = \left(\frac{m_x}{m_x + m_n}\right) Q}$$

1.1 (b)	$E_n = \left(\frac{m_x}{m_x + m_n}\right) Q$	0.6 pts

1.1 (c) From the expression obtained in 1.1 (b) and the Q-values calculated from 1.1 (a), the energy of neutrons produced from D-D reactions is:

$$E_n = \left(\frac{m_{He3}}{m_{He3} + m_n}\right) Q$$

$$E_n = \left(\frac{3.016029}{3.016029 + 1.008665}\right) 3.270 \text{ MeV} = \mathbf{2.450\ MeV}$$

While neutrons for the D-T reaction will have an energy of:

$$E_n = \left(\frac{m_{He4}}{m_{He4} + m_n}\right) Q$$

$$E_n = \left(\frac{4.002603}{4.002603 + 1.008665}\right) 3.270 \text{ MeV} = \mathbf{14.05\ MeV}$$

1.1 (c)	D-D: $E_n = 2.450$ MeV D-T: $E_n = 14.05$ MeV	0.1 pt 0.1 pt

1.2 (a)

For the D-T reaction:

Total mass of reactants: $m_D + m_T = 2.014102 \text{ u} + 3.016049 \text{ u} = 5.030151 \text{ u}$

Given the Q-value calculated from **1.1 (b)**, $Q_{DT} = 17.59$ MeV, the energy per unit mass of D-T fusion reactants is then:

$$\frac{E}{m} = \frac{17.59 \text{ MeV}}{5.030151 \text{ u}}$$

$$= 3.496913 \frac{\text{MeV}}{\text{u}} \left(\frac{1 \text{ kWh}}{2.24694 \times 10^{19} \text{ MeV}}\right)\left(\frac{1 \text{ u}}{1.660565 \times 10^{-27} \text{kg}}\right)$$

$$= 9.372 \times 10^7 \text{ kWh/kg}$$

For a fusion plant with 35% efficiency, the extracted energy per unit mass is just:

$$0.35\left(\frac{E}{m}\right) = 0.35(9.372 \times 10^7 \text{ kWh/kg}) = 3.280 \times 10^7 \text{ kWh/kg}$$

Then for $E_1 = 3600$ kWh, the corresponding D-T reactant mass is:

$$m_1 = \frac{E_1}{\frac{E}{m}} = \frac{3{,}600 \text{ kWh}}{3.280 \times 10^7 \text{ kWh/kg}} = \mathbf{1.097 \times 10^{-4} \text{ kg}}$$

For the D-D reaction:

Total mass of reactants: $2 \cdot m_D = 2 \cdot 2.014102 \text{ u} = 4.028204 \text{ u}$

Given the Q-value calculated from **1.1 (b)**, $Q_{DD} = 3.270 \text{ MeV}$, the energy per unit mass of DD fusion reactants is then:

$$\frac{E}{m} = \frac{3.270 \text{ MeV}}{4.028204 \text{ u}}$$

$$= 0.811776 \frac{\text{MeV}}{\text{u}} \left(\frac{1 \text{ kWh}}{2.24694 \times 10^{19} \text{ MeV}}\right)\left(\frac{1 \text{ u}}{1.660565 \times 10^{-27} \text{kg}}\right)$$

$$= 2.176 \times 10^7 \text{ kWh/kg}$$

For a fusion plant with 35% efficiency, the extracted energy per unit mass is just:

$$0.35\left(\frac{E}{m}\right) = 0.35(2.176 \times 10^7 \text{ kWh/kg}) = 7.615 \times 10^6 \text{ kWh/kg}$$

Then for $E_1 = 3{,}600 \text{ kWh}$, the corresponding DD reactant mass is:

$$m_1 = \frac{E_1}{\frac{E}{m}} = \frac{3{,}600 \text{ kWh}}{7.615 \times 10^6 \text{ kWh/kg}} = \mathbf{4.728 \times 10^{-4} \text{ kg}}$$

1.2 (a)	D-T:	1.098×10^{-4} kg	$(1.087 \times 10^{-4} - 1.108 \times 10^{-4} \text{ kg})$	0.25 pts
	D-D:	4.728×10^{-4} kg	$(4.680 \times 10^{-4} - 4.775 \times 10^{-4} \text{ kg})$	0.25 pts
	Note: Answers within $\pm 1\%$ of these values are acceptable.			

1.2 (b) Given the following values:

$E_f = 200 \text{ MeV}$

$m_{react} = m_{U235} + m_{neutron} = 235.044 \text{ u} + 1.008665 \text{ u} = 236.053 \text{ u}$

$$\frac{E}{m} = \frac{200 \text{ MeV}}{236.053 \text{ u}}$$

$$= 0.847269 \frac{\text{MeV}}{\text{u}} \left(\frac{1 \text{ kWh}}{2.24694 \times 10^{19} \text{ MeV}}\right)\left(\frac{1 \text{ u}}{1.660565 \times 10^{-27} \text{kg}}\right)$$

$$= 2.271 \times 10^7 \text{ kWh/kg}$$

For a fission plant with 30% efficiency, the extracted energy per unit mass is just:

$$0.30\left(\frac{E}{m}\right) = 0.30(2.271 \times 10^{7}\ \text{kWh/kg}) = 6.812 \times 10^{6}\ \text{kWh/kg}$$

Then for $E_1 = 3600\ \text{kWh}$, the corresponding fission reactant mass is:

$$m_1 = \frac{E_1}{\frac{E}{m}} = \frac{3600\ \text{kWh}}{6.812 \times 10^{6}\ \text{kWh/kg}} = \mathbf{5.285 \times 10^{-4}\ kg}$$

1.2 (b)	5.285×10^{-4} kg ($5.232 \times 10^{-4} - 5.337 \times 10^{-4}$ kg) *Note: Answers within ±1% of this value are acceptable.*	0.25 pts

1.2 (c) For bituminous coal:

$$\frac{E}{m} = 31\frac{\text{kJ}}{\text{g}}\left(\frac{1\ \text{eV}}{1.60218\ \times 10^{-22}\ \text{kJ}}\right)\left(\frac{1\ \text{MeV}}{1 \times 10^{6}\ \text{eV}}\right)\left(\frac{1\ \text{kWh}}{2.24694 \times 10^{19}\ \text{MeV}}\right)\left(\frac{1{,}000\ \text{g}}{1\ \text{kg}}\right)$$
$$= 8.611\ \text{kWh/kg}$$

Then for $E_1 = 3600\ \text{kWh}$, the corresponding coal mass is:

$$m_1 = \frac{E_1}{\frac{E}{m}} = \frac{3600\ \text{kWh}}{8.611\ \text{kWh/kg}} = \mathbf{4.181 \times 10^{2}\ kg}$$

1.2 (c)	4.181×10^{2} kg	0.15 pt

$$\frac{4.181 \times 10^{2}\ \text{kg}}{1.098 \times 10^{-4}\ \text{kg}} = \mathbf{3.808 \times 10^{6}}$$

1.2 (d)	3.808×10^{6} ($3.769 \times 10^{6} - 3.846 \times 10^{6}$) *Note: Answers within ±1% of this value are acceptable.*	0.1 pt

Part 2. Tritium production (2.0 pts)

2.1 Given the nuclear reaction:

$${}^{7}_{3}\text{Li} + {}^{1}_{0}\text{n} \rightarrow {}^{4}_{2}\text{He} + {}^{3}_{1}\text{H} + {}^{1}_{0}\text{n}$$

and the expression for conservation of energy:

$$E_{Li} + E_n \rightarrow E_{He} + E_H + E_{n\prime} \qquad (1)$$

where the energy of the particle i is the sum of its kinetic and rest mass energies:

$$E_i = KE_i + m_i c^2 \qquad (2)$$

The minimum incident neutron energy or its threshold energy is calculated by assuming that the kinetic energies of the reaction products are zero. Assuming also that the target Li-7 is at rest, equation (1) becomes:

$$m_{Li}c^2 + \frac{1}{2}m_n v_n^2 + m_n c^2 = m_{He}c^2 + m_H c^2 + m_n c^2 \qquad (3)$$

Combining all rest energy terms in the RHS:

$$\frac{1}{2}m_n v_n^2 = -(m_{Li}c^2 + m_n c^2) + (m_{He}c^2 + m_H c^2 + m_n c^2) \qquad (4)$$

$$\frac{1}{2}m_n v_n^2 = -[(m_{Li} + m_n) - (m_{He} + m_H + m_n)]c^2 \qquad (5)$$

$$KE_n = -[(m_{Li} + m_n) - (m_{He} + m_H + m_n)]c^2 \qquad (6)$$

The RHS of equation (6) is the negative of the Q-value of the reaction, hence:

$$KE_n = -Q$$

Therefore, the minimum incident energy is determined from the Q-value for the reaction:

$$KE_n = -Q = -[(m_{Li} + m_n) - (m_{He} + m_H + m_n)]c^2$$

$$KE_n = -[(7.016003\text{ u} + 1.008665\text{ u}) - (4.002603\text{ u} + 3.016029\text{ u} + 1.008665\text{ u})]\ (931.5\text{ MeV/u})$$

$$KE_n = -(-0.002649)\left(931.5\frac{\text{MeV}}{\text{u}}\right) = \mathbf{2.468\ MeV}$$

2.1	2.468 MeV (2.443 – 2.493 MeV) *Note: Answers within ±1% of this value are acceptable.*	0.8 pts

2.2 (a) Combining the following equations:

$R = \sigma N \phi V$ and $N = \frac{\rho N_A}{m} = \frac{M N_A}{V m}$; where $\rho = \frac{M}{V}$

We get: $\boldsymbol{R = \frac{\sigma M N_A \phi}{m}}$

Given the following:

$M_{Li} = 1{,}000\text{ g}$

$N_A = 6.02214 \times 10^{23}\text{ mol}^{-1}$

$\sigma = 940\text{ barn} = 9.4 \times 10^{-22}\text{cm}^2$

$\phi = 3.5 \times 10^{14}\text{ cm}^{-2}\text{s}^{-1}$

$m_{Li6} = 6.015122\text{ u or g/mol}$

We calculate the mass of Li-6 from 1000 g of natural Lithium.

$$m_{Li6} = 1000\text{ g} \times \frac{1\text{ mol Lithium}}{0.0485 \times 6.015122\text{ g} + 0.9515 \times 7.016003\text{ g}} \times 0.0485 \times \frac{6.015122\text{ g Li} - 6}{1\text{ mol Li} - 6} = 41.87084\text{ g}$$

Applying the formula gives:

$$R = \frac{MN_A\sigma\phi}{m} = \frac{(41.87084\text{ g})(6.02214 \times 10^{23}\text{ mol}^{-1})(9.4 \times 10^{-22}\text{cm}^2)(3.5 \times 10^{14}\text{ cm}^{-2}\text{s}^{-1})}{6.015122\text{ g/mol}} = 1.3791588 \times 10^{18}\text{ s}^{-1}$$

In 24 hrs (86,400 s), the total ${}^3_1\text{H}$ nuclides produced is then:

$$\#\,{}^3_1\text{H} = 1.3791588 \times 10^{18}\text{ s}^{-1}(86{,}400\text{ s}) = 1.1915932 \times 10^{23}\text{ nuclides}$$

Since $m_{H3} = 3.016049$ u or g/mol, then the mass in grams is:

$$M_{H3} = \frac{\left(\#\,{}^3_1\text{H}\right)(m_{H3})}{N_A} = \frac{(1.1915932 \times 10^{23})(3.016049\text{ g/mol})}{6.02214 \times 10^{23}\text{ mol}^{-1}} = \mathbf{0.597\text{ g}}$$

2.2 (a)	0.597 g of ${}^3_1\text{H}$ (0.590 – 0.603 g) *Note: Answers within $\pm 1\%$ of this value are acceptable.*	1.0 pt

2.2 (b) By ratio and proportion, we obtain:

$$M'_{Li} = M'_{H3}\left(\frac{M_{Li}}{M_{H3}}\right) = 1.50 \times 10^5\text{ g}\left(\frac{1\times10^3\text{g}}{0.597\text{ g}}\right) = \mathbf{2.513 \times 10^5\text{ kg}}$$

2.2 (b)	2.513×10^5 kg ($2.488 \times 10^5 - 2.538 \times 10^5$ kg) *Note: Answers within $\pm 1\%$ of this value are acceptable.*	0.2 pts

Part 3. Overcoming the Coulomb barrier (2.5 pts)

3.1 (a) We first calculate the distance between two deuterons using the empirical formula for the radius of the nuclei: $r = r_0 A^{1/3}$

Given that $A_{H2} = 2$:

$$d_{DD} = 2r = 2 \cdot r_0 A^{1/3} = (2)(1.2 \times 10^{-15}\text{m})(2^{1/3}) = 3.024 \times 10^{-15}\text{ m}$$

Then, given $q = 1.60218 \times 10^{-19}$ C, $k_e = 8.98755 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2$

$$U = \frac{k_e q^2}{d_{DD}} = \frac{\left(8.98755 \times 10^9\text{ N}\cdot\frac{\text{m}^2}{\text{C}^2}\right)(1.60218 \times 10^{-19}\text{ C})^2}{3.024 \times 10^{-15}\text{ m}} = \mathbf{7.630 \times 10^{-14}\text{ N}\cdot\text{m}}$$

3.1 (a)	7.630×10^{-14} N·m ($7.553 \times 10^{-14} - 7.706 \times 10^{-14}$ N·m)	0.8 pts

	Note: Answers within $\pm 1\%$ of this value are acceptable. Alternate unit is joules or J.

3.1 (b) We first calculate the distance between a deuteron and a triton Given that $A_{H2} = 2$ and $A_{H3} = 3$:

$$d_{DT} = r_{H2} + r_{H3} = r_0\left(A_{H2}^{1/3} + A_{H3}^{1/3}\right) = (1.2 \times 10^{-15}\,\mathrm{m})(2^{1/3} + 3^{1/3}) = 3.243 \times 10^{-15}\ \mathrm{m}$$

Then, given $q = 1.60218 \times 10^{-19}\ \mathrm{C}$, $k_e = 8.98755 \times 10^{9}\ \mathrm{N \cdot m^2/C^2}$

$$U = \frac{k_e q^2}{d_{DT}} = \frac{\left(8.98755 \times 10^9\ \mathrm{N} \cdot \frac{\mathrm{m}^2}{\mathrm{C}^2}\right)(1.60218 \times 10^{-19}\ \mathrm{C})^2}{3.243 \times 10^{-15}\ \mathrm{m}} = \mathbf{7.115 \times 10^{-14}\ N \cdot m}$$

3.1 (b)	$7.115 \times 10^{-14}\ \mathrm{N \cdot m}$ $(7.044 \times 10^{-14} - 7.186 \times 10^{-14}\ \mathrm{N \cdot m})$ *Note: Answers within $\pm 1\%$ of this value are acceptable. Alternate unit is joules or J.*	0.8 pts

3.2 (a) From equation (5), the expression for temperature with $\bar{E} = \frac{1}{2}U$ is:

$$T = \frac{2\bar{E}}{3k_B} = \frac{U}{3k_B}$$

Given $1\ \mathrm{J} = 6.24150 \times 10^{18}\ \mathrm{eV}$ and $k_B = 8.61733 \times 10^{-5}\ \mathrm{eV/K}$

For DD: $U = 7.628 \times 10^{-14}\ \mathrm{J}\left(\frac{6.24150 \times 10^{18}\ \mathrm{eV}}{1\mathrm{J}}\right) = 4.762 \times 10^{6}\mathrm{eV}$

$$T = \frac{4.762 \times 10^6\mathrm{eV}}{3(8.61733 \times 10^{-5}\ \mathrm{eV/K})} = \mathbf{1.842 \times 10^9\ K}$$

For DT: $U = 7.114 \times 10^{-13}\ \mathrm{J}\left(\frac{6.24150 \times 10^{18}\ \mathrm{eV}}{1\mathrm{J}}\right) = 4.440 \times 10^{6}\mathrm{eV}$

$$T = \frac{4.440 \times 10^6\mathrm{eV}}{3(8.61733 \times 10^{-5}\ \mathrm{eV/K})} = \mathbf{1.718 \times 10^9\ K}$$

3.2 (a)	DD: $1.842 \times 10^9\ \mathrm{K}$ $(1.824 \times 10^9 - 1.860 \times 10^9\ \mathrm{K})$ DT : $1.718 \times 10^9\ \mathrm{K}$ $(1.701 \times 10^9 - 1.735 \times 10^9\ \mathrm{K})$ *Note: Answers within $\pm 1\%$ of these values are acceptable.*	0.3 pts 0.3 pts

Given the actual ignition temperature for D-D: $4.5 \times 10^8\ \mathrm{K}$, the % difference is:

$$\%diff = \frac{|4.5 \times 10^8\ \mathrm{K} - 1.842 \times 10^9 \mathrm{K}|}{\frac{4.5 \times 10^8\ \mathrm{K} + 1.842 \times 10^9 \mathrm{K}}{2}} \times 100 = 121.5\%$$

Given the actual ignition temperature for D-D: 4.5×10^8 K, the % difference is:

$$\%diff = \frac{|1.5 \times 10^8\ \mathrm{K} - 1.718 \times 10^9 \mathrm{K}|}{\frac{1.5 \times 10^8\ \mathrm{K} + 1.718 \times 10^9 \mathrm{K}}{2}} \times 100 = 167.9\%$$

3.2 (b)	DD: 121.5% $(120.3 - 122.7\%)$	0.15 pts
	DT : 167.9% $(166.2 - 169.6\%)$	0.15 pts
	Note: Answers within $\pm 1\%$ of these values are acceptable	

Part 4. Portable neutron generators (3.5 pts)

4.1 The surface density (N_s) is related to the atom number density (N) as follows:

$$N_s = N \cdot l = \frac{\rho N_A}{m} \cdot l$$

where l is the sample thickness, which is given in the problem (0.1 cm).

For $TiD_{1.64}$, given $\rho = 3.92\ \mathrm{g/cm^3}$ and $m = 51.170127$ u, the surface density is:

$$N_s = N \cdot l = \frac{\rho N_A}{m} \cdot l = \frac{3.92 \frac{\mathrm{g}}{\mathrm{cm^3}} (6.02214 \times 10^{23}\ \mathrm{mol^{-1}})}{51.170127 \frac{\mathrm{g}}{\mathrm{mol}}} \cdot (0.1\ \mathrm{cm}) = 4.613 \times 10^{21}\ \mathrm{cm^{-2}}$$

Multiplying the value by 1.64 gives the number of D atom per unit area:

$$N_{s,D} = 1.64 \cdot 4.613 \times 10^{21}\ \mathrm{cm^{-2}} = \mathbf{7.566 \times 10^{21}\ cm^{-2}}$$

For $TiT_{1.64}$, given $\rho = 4.03\ \mathrm{g/cm^3}$ and $m = 52.813320$ u, the surface density is:

$$N_s = N \cdot l = \frac{\rho N_A}{m} \cdot l = \frac{4.03 \frac{\mathrm{g}}{\mathrm{cm^3}} (6.02214 \times 10^{23}\ \mathrm{mol^{-1}})}{52.813320 \frac{\mathrm{g}}{\mathrm{mol}}} \cdot (0.1\ \mathrm{cm}) = 4.595 \times 10^{21}\ \mathrm{cm^{-2}}$$

Multiplying the value by 1.64 gives the number of T atom per unit area:

$$N_{s,T} = 1.64 \cdot 4.595 \times 10^{21}\ \mathrm{cm^{-2}} = \mathbf{7.536 \times 10^{21}\ cm^{-2}}$$

4.1	(a) D: $N_{s,D} = 7.566 \times 10^{21}\ \text{cm}^{-2}$ $(7.490 \times 10^{21} - 7.642 \times 10^{21}\ \text{cm}^{-2})$ (b) T: $N_{s,T} = 7.536 \times 10^{21}\ \text{cm}^{-2}$ $(7.461 \times 10^{21} - 7.612 \times 10^{21}\ \text{cm}^{-2})$ *Note: Answers within ±1% of these values are acceptable*	0.7 pts 0.7 pts

4.2 By combining the following equations:

$$S = N_s \sigma \varphi$$

$$\varphi = \frac{I}{q} \cdot a$$

The expression for the deuteron beam current can be written as:

$$I = \frac{Sq}{N_s \sigma a}$$

Given the following:

Neutron generator parameter: $a = 6.25 \times 10^{-4}$

Deuteron charge: $q = 1.60218 \times 10^{-19}\ \text{C}$

$S = 1 \times 10^7 \text{n/s}$

From 4.1(a), $N_{s,D} = 7.566 \times 10^{21}\ \text{cm}^{-2}$

For D-D: $\sigma_{D-D} = 16.9\ \text{mb} = 1.69 \times 10^{-26}\ \text{cm}^2$

$$I = \frac{Sq}{N_{s,D} \sigma_{D-D} a}$$

$$I = \frac{(1 \times 10^7 \text{s}^{-1})(1.60218\ \times 10^{-19}\ \text{C})}{(7.566 \times 10^{21}\ \text{cm}^{-2})(1.69 \times 10^{-26}\ \text{cm}^2)(6.25 \times 10^{-4})} = \mathbf{2.005 \times 10^{-5} A}$$

For D-T: $\sigma_{D-T} = 293 \cdot 16.9\ \text{mb} = 4.9517 \times 10^{-24}\ \text{cm}^2$

$$I = \frac{Sq}{N_{s,T} \sigma_{D-T} a}$$

$$I = \frac{(1 \times 10^7 \text{s}^{-1})(1.60218\ \times 10^{-19}\ \text{C})}{(7.536 \times 10^{21}\ \text{cm}^{-2})(4.9517 \times 10^{-24}\ \text{cm}^2)(6.25 \times 10^{-4})} = \mathbf{6.869 \times 10^{-8} A}$$

4.2	D-D: $2.005 \times 10^{-5}\text{A}$ $(1.985 \times 10^{-5} - 2.025 \times 10^{-5}\ \text{A})$ D-T : $6.869 \times 10^{-8}\text{A}$ $(6.801 \times 10^{-8} - 6.938 \times 10^{-8}\ \text{A})$ *Note: Answers within ±1% of these values are acceptable.*	0.7 pts 0.7 pts

4.3 (a)

$$\phi = \frac{S}{4\pi r^2} = \frac{1 \times 10^7 \text{n/s}}{(4)(\pi)(3\text{ cm})^2} = \mathbf{8.842 \times 10^4\ n/cm^2 s}$$

4.3 (a)	$8.842 \times 10^4\ \text{n/cm}^2 \cdot \text{s}$ $(8.754 \times 10^4 - 8.930 \times 10^4\ \text{n/cm}^2 \cdot \text{s})$ *Note: Answer within $\pm 1\%$ of this value is acceptable.*	0.2 pts

4.3 (b)

Combining the expression for emerging flux at the collimator outlet:

$$\phi = \frac{\phi_0 A}{4\pi L^2}$$

And the area for a circular collimator:

$$A = \frac{\pi D^2}{4}$$

We obtain:

$$\phi = \frac{\phi_0 D^2}{16L^2}$$

The ratio of the fluxes at the outlet apertures of the two collimators

$$\frac{\phi_1}{\phi_2} = \frac{\frac{\phi_0 {D_1}^2}{16L^2}}{\frac{\phi_0 {D_2}^2}{16L^2}} = \frac{{D_1}^2}{{D_2}^2}$$

4.3 (b)	$\phi_2 = \frac{{D_2}^2}{{D_1}^2}\phi_1 = \left(\frac{D_2}{D_1}\right)^2 \phi_1$	0.5 pts

Q2 STERILE INSECT TECHNIQUE FOR MOSQUITOES (10 pts)

Part 1. Absorbed Dose Measurement Using Fricke Dosimeter (4.5 points)

1.1 The power can be expressed as the product of activity and energy

$$P\ (MeV\ s^{-1}) = A \times E(MBq \cdot MeV) \times \frac{10^6 Bq}{1\ MBq} \times \frac{1\ s^{-1}}{1\ Bq}$$

$$P\ (MeV\ s^{-1}) = 10^6\ AE$$

We are asked to express it in $J\ h^{-1}$:

$$P\ (J\ h^{-1}) = 10^6\ AE\ \left(\frac{MeV}{s}\right) \times \left(\frac{10^6 eV}{1\ MeV}\right) \times \left(\frac{1\ J}{6.2415\ \times\ 10^{18} eV}\right) \times \left(\frac{3600\ s}{1\ h}\right)$$

$$P\ (J\ h^{-1}) = 5.768 \times 10^{-4} AE$$

1.1	$P\ (J\ h^{-1}) = 5.768 \times 10^{-4} AE$	0.5 pt.

1.2 Because the gamma source is isotropic, the energy that is emitted over the surface of the sphere of radius r meters, which has an area of $4\pi r^2$ is:

$$P\ (J\ h^{-1} m^{-2}) = \frac{5.768 \times 10^{-4} AE}{4\pi r^2}$$

$$P\ (J\ h^{-1} m^{-2}) = \frac{1.442 \times 10^{-4} AE}{\pi r^2}$$

1.2	$P\ (J\ h^{-1} m^{-2}) = \frac{1.442 \times 10^{-4} AE}{\pi r^2}\ \ or\ \ \frac{4.590 \times 10^{-5} AE}{r^2}$	1 pt.

1.3 The activity is $8000\ Ci$, and this is equivalent to $2.96 \times 10^8\ MBq$, while the mass attenuation coefficient, in $\frac{m^2}{kg}$, is 7.2×10^{-3}. To determine the dose rate:

$$DR\ (Gy\ h^{-1}) = \frac{1.442 \times 10^{-4} AE}{\pi r^2} \left(\frac{\mu}{\rho}\right)$$

$$DR\ (Gy\ h^{-1}) = \frac{1.442 \times 10^{-4} \times 2.96 \times 10^8 \times 1.25}{\pi \times 0.5^2} (7.2 \times 10^{-3})$$

$$DR\ (Gy\ h^{-1}) = 489.11\ Gy\ h^{-1}$$

In 10 minutes, the total dose absorbed is

$$D\ (Gy) = 489.11\ \frac{Gy}{h} \times 10\ min \times \frac{1\ h}{60\ min}$$

$$D\ (Gy) = 81.52\ Gy$$

1.3	$DR\ (Gy\ h^{-1}) = 489.1\ Gy\ h^{-1}$	1 pt
	$D\ (Gy) = 81.52\ Gy$	0.5 pt

1.4

$$D = \frac{\Delta OD}{\varepsilon G(Fe^{3+})\rho l}$$

$$G(Fe^{3+}) = \frac{\Delta OD}{\varepsilon D \rho l}$$

$$G(Fe^{3+}) = \frac{(0.184 - 0.003)}{2174\frac{L}{mol\ cm} \times 81.5\frac{J}{kg} \times 1.024\frac{g}{cm^3} \times 1cm \times \frac{1000cm^3}{1L} \times \frac{1kg}{1000g} \times \frac{1mol}{10^6\mu mol}}$$

$$G(Fe^{3+}) = 0.9974\ \mu mol\ J^{-1}$$

1.4	$G(Fe^{3+}) = 0.9974\ \mu mol\ J^{-1}$	0.5 pt.

1.5 Given that all the Co-60 gamma ray photons deposit 300 keV into the Fricke dosimeter per interaction. The number of interactions needed to provide the required absorbed doses can be calculated from the total energy deposited to the dosimeter and the energy deposited per interaction

$$No. of\ interactions = \frac{total\ energy\ deposited}{energy\ deposited\ per\ interaction}$$

For 10 Gy,

$$No. of\ photon\ interactions = \frac{10\ \frac{J}{kg} \times 1g \times \frac{1kg}{1000g}}{300\frac{keV}{interaction} \times \frac{1000\ eV}{1\ keV} \times \frac{1J}{6.2415 \times 10^{18}eV}}$$

$$\boldsymbol{No. of\ photon\ interactions = 2.0805 \times 10^{11}}$$

Following the same calculation,

Absorbed dose	No. of photon interactions
10	2.081×10^{11}
20	4.161×10^{11}
30	6.242×10^{11}
40	8.322×10^{11}
50	1.040×10^{12}

1.5

Absorbed dose	No. of photon interactions
10	2.081×10^{11}
20	4.161×10^{11}
30	6.242×10^{11}
40	8.322×10^{11}
50	1.040×10^{12}

1 pt. (0.2 pt. each)

Part 2. Egg Hatch and Dose in SIT (3.5 points)

2.1. Given the equation for egg hatch and observed hatch of 40%, find the dose.

$$\text{Egg hatch } (\%) = 85\% \times \exp(-0.065 \text{ Dose})$$

$$40\% = 85\% \times \exp(-0.065 \text{ Dose})$$

$$\text{Dose} = \frac{\ln(40 \div 85)}{-0.065}$$

2.1 $\text{Dose} = 11.60\ Gy$ 0.5 pt.

2.2 The dose 11.60 Gy with 40% egg hatch was administered for 10 minutes, find the time it took for irradiation to result to 1% egg hatch.

$$\text{Egg hatch } (\%) = 85\% \times \exp(-0.065 \text{ Dose})$$

$$1\% = 85\% \times \exp(-0.065 \text{ Dose})$$

$$\text{Dose}_{\text{new}} = \frac{\ln(1 \div 85)}{-0.065}$$

$$\text{Dose}_{\text{new}} = 68.35\ \text{Gy}$$

$$\text{Time}_{\text{new}} = (10\ \text{min}) \times \left(\frac{68.35\ \text{Gy}}{11.60\ \text{Gy}}\right)$$

2.2	Time_{new} = 58.94 min or 58 min & 56 sec.	1.0 pt.

2.3 Given the equation for the shielded dose, calculate the required lead thickness to increase the egg hatch from 0.1% to 2%. Use a mass attenuation coefficient, $\frac{\mu}{\rho}$, of 0.058 cm²/g for these rays in lead (ρ=11.3 g/cm³).

$$D_0 = \frac{\ln(0.1 \div 85)}{-0.065} = 103.77\ \text{Gy}$$

$$D_i = \frac{\ln(2 \div 85)}{-0.065} = 57.68\ \text{Gy}$$

$$D_i = D_0 \exp\left(-\left(\frac{\mu}{\rho}\right)\rho x\right)$$

$$x = \frac{\ln(D_0 \div D_i)}{\left(\frac{\mu}{\rho}\right)\rho} = \frac{\ln(103.77 \div 57.68)}{(0.058\ \text{cm}^2/\text{g})(11.3\ \text{g/cm}^3)}$$

2.3	x = 0.8960 cm	1.0 pt.

2.4 (a) Calculate the effective mass attenuation coefficient of stainless steel.

$$\left(\frac{\mu}{\rho}\right)_{ss} = \sum_i w_i \times \left(\frac{\mu}{\rho}\right)_i$$

$$\left(\frac{\mu}{\rho}\right)_{ss} = (0.0008)(0.0568) + (0.02)(0.0521) + (0.00045)(0.0551) + (0.0003)(0.0568) + (0.01)(0.0567) + (0.19)(0.0528) + (0.095)(0.0548) + (0.68345)(0.0534) = 0.05343$$

2.4 (a)	0.05343 cm²/g	0.5 pt.

2.4 (b) Calculate how many 1-mm thick stainless-steel sheets are needed to achieve the desired shielding effect that keeps the egg hatch rate close to 2%.

$$x = \frac{\ln(D_0 \div D_i)}{\left(\frac{\mu}{\rho}\right)\rho} = \frac{\ln(103.77 \div 57.68)}{(0.05343\ \text{cm}^2/\text{g})(8.03\ \text{g/cm}^3)} = 1.369\ \text{cm} = 13.69\ \text{mm}$$

2.4 (b)	14 sheets	0.5 pt.

Part 3. Competitiveness and Dose in SIT (2.0 points)

3.1 Given the equation for male competitiveness as a function of dose, find C when dose is 40 Gy.

$$\text{C} = \frac{1}{1 + \exp(-4.7\ + [3.6 \times \log(Dose)])}$$

$$\text{C} = \frac{1}{1 + \exp(-4.7\ + [3.6 \times \log(40)])}$$

3.1	C = 0.2559	1.0 pt.

3.2 Given that a competitiveness (C) of 0.5 is desired, find the irradiation dose required to achieve this.

$$\text{C} = \frac{1}{1 + \exp(-4.7\ + [3.6 \times \log(\text{Dose})])}$$

$$1 + \exp(-4.7\ + [3.6 \times \log(\text{Dose})]) = \frac{1}{\text{C}}$$

$$\exp(-4.7\ + [3.6 \times \log(\text{Dose})]) = \frac{1}{\text{C}} - 1$$

$$-4.7\ + [3.6 \times \log(\text{Dose})] = \ln\left(\frac{1}{\text{C}} - 1\right)$$

$$3.6 \times \log(\text{Dose}) = \ln\left(\frac{1}{\text{C}} - 1\right) + 4.7$$

$$\log(\text{Dose}) = \frac{\left[\ln\left(\frac{1}{\text{C}} - 1\right) + 4.7\right]}{3.6}$$

$$\text{Dose} = 10\hat{}\ \frac{\left[\ln\left(\frac{1}{\text{C}} - 1\right) + 4.7\right]}{3.6}$$

$$\text{Dose} = 10\hat{}\ \frac{\left[\ln\left(\frac{1}{0.5} - 1\right) + 4.7\right]}{3.6}$$

3.2	Dose = 20.21 Gy	1.0 pt.

Q3 PARTICLES FOR DESTROYING CANCER (10 pts)

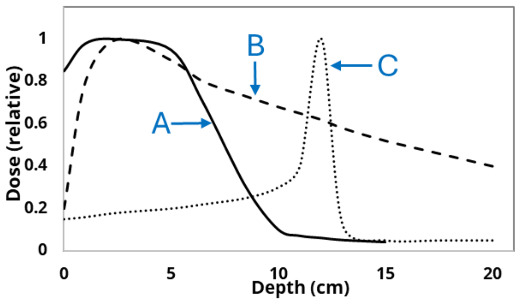
Part 1. Proton Stopping Power, Range, and Dose (5.0 pts)

1.1

1.1	Stopping power is defined as the <u>rate of energy loss per unit path length by a particle traveling through a material</u>.	0.2 pts
	LET is the average <u>energy deposited/transferred to the material per unit path length</u>.	0.2 pts

1.2

1.2



A:	Electron	0.2 pts
B:	Photon	0.2 pts
C:	Proton	0.2 pts

1.3

The Bethe-Bloch equation for the mass stopping power of protons at 1-200 MeV energy range is given by:

$$\frac{S}{\rho} = -\frac{dE}{\rho dx} = \frac{5.08 \times 10^{-31} z^2 n}{\rho {\beta_v}^2}[F(\beta_v) - ln\,(I)]$$

where:

β_v is the velocity of the incident particle v relative to the speed of light c
I is the excitation energy of a target material (for water, $I = 74.6\ eV$)
z is the charge of the incident particle (for proton, $z = +1$)
n is the number of electrons in a material per unit volume (in m^3) calculated using the following equation:

$$n = \frac{N_a Z \rho}{A}$$

where:

N_A is the Avogadro's number ($N_A = 6.02214 \times 10^{23}\ mol^{-1}$)

ρ is the density of a material (for water, $\rho = 1.000\ g \cdot cm^{-3}$)

Z/A is the ratio of atomic number to the mass number of a material (for water, $\frac{Z}{A} = \frac{1+1+8}{1+1+16} = \frac{10}{18} = 0.555555555$)

$$n = \frac{N_a \rho Z}{A} = (6.02214 \times 10^{23})(1\ \ g \cdot cm^{-3})(0.55556) \times \frac{100^3\ cm^3}{1^3\ m^3} = 3.345633333 \times 10^{29}$$

$$\frac{S}{\rho} = -\frac{dE}{\rho dx} = \frac{(5.08 \times 10^{-31})(1^2)(3.345633333 \times 10^{29})}{\rho {\beta_v}^2}[F(\beta_v) - ln(74.6)]$$
$$= \frac{0.169958173}{\rho {\beta_v}^2}[F(\beta_v) - 4.312140507]$$

1.3	$$\frac{S}{\rho} = -\frac{dE}{\rho dx} = \frac{0.170}{\rho {\beta_v}^2}[F(\beta_v) - 4.31]$$ Other acceptable answer: $$\frac{S}{\rho} = -\frac{dE}{\rho dx} = \frac{0.17}{\rho {\beta_v}^2}[F(\beta_v) - 4.31]$$ $$\frac{S}{\rho} = -\frac{dE}{\rho dx} = \frac{0.17}{(\ g \cdot cm^{-3})\ {\beta_v}^2}[F(\beta_v) - 4.31]$$	0.5 pts

1.4 (a)

Step 1: Solving for total energy $\boldsymbol{E}$, use the equation:

$$\boldsymbol{E \equiv E_K + E_0 = E_K + m_0 c^2}$$

where:

E is the total energy or the sum of rest-mass energy and kinetic energy

E_K is the kinetic energy of a particle (at 1, 10, 100 MeV)

E_0 is the rest-mass energy of a particle (for proton, $E_0 = 938.27209\ MeV = 1.503280777 \times 10^{-10}\ kg \cdot m^2 \cdot s^{-2}$)

m_0 is the rest-mass of a particle (for proton, $m_0 = 1.67262 \times 10^{-27}\ kg$)

c is the speed of light ($c = 299\ 792\ 458\ m \cdot s^{-1}$)

Conversion:

$1\ MeV = 1 \times 10^6\ electon\ volts\ (eV) = 1.60218 \times 10^{-13}\ joules\ (J) or\ kg \cdot m^2 \cdot s^{-2}$

$1\ eV = 1.60218 \times 10^{-19}\ joules\ (J)\ or\ kg \cdot m^2 \cdot s^{-2}$

Solving for total energy, E

At $E_K = 1\ MeV$, $E = 1\ MeV + 938.27209\ MeV = 939.27209\ MeV = 1.504882957 \times 10^{-10}\ kg \cdot m^2 \cdot s^{-2}$

At $E_K = 10\ MeV$, $E = 10\ MeV + 938.27209\ MeV = 948.27209\ MeV = 1.519302577 \times 10^{-10}\ kg \cdot m^2 \cdot s^{-2}$

At $E_K = 100\ MeV$, $E = 100\ MeV + 938.27209\ MeV = 1038.27209\ MeV = 1.663498777 \times 10^{-10}\ kg \cdot m^2 \cdot s^{-2}$

Step 2: Using the relativistic energy-momentum relation for a particle:

$$\boldsymbol{E^2 = (m_0c^2)^2 + (pc)^2}$$

To solve for pc: $\boldsymbol{pc = \sqrt{E^2 - E_0^2}}$

At $E = 939.27208943\ MeV = 1.504879791 \times 10^{-10}\ kg \cdot m^2 \cdot s^{-2}$, $pc = 43.33063789\ MeV = 6.942348142 \times 10^{-12}\ kg \cdot m^2 \cdot s^{-2}$

At $E = 948.27208943\ MeV = 1.519299381 \times 10^{-10}\ kg \cdot m^2 \cdot s^{-2}$, $pc = 137.3515264\ MeV = 2.200618685 \times 10^{-11}\ kg \cdot m^2 \cdot s^{-2}$

At $E = 1038.27208943\ MeV = 1.663495277 \times 10^{-10}\ kg \cdot m^2 \cdot s^{-2}$, $pc = 444.5834207\ MeV = 7.123026650 \times 10^{-11}\ kg \cdot m^2 \cdot s^{-2}$

Step 3: Using the equation: $\boldsymbol{\beta_v \equiv \frac{v}{c} = \frac{pc}{E}}$ to solve for velocity: $\boldsymbol{v = \frac{pc^2}{E}}$

where:

v is the velocity of a particle

p is the momentum of a particle

At $pc = 43.33063788\ MeV$, $v = 0.046132147c = 13{,}830{,}070\ m \cdot s^{-1}$

At $pc = 137.3515263\ MeV$, $v = 0.144844004c = 43{,}423{,}140\ m \cdot s^{-1}$

At $pc = 444.5834206\ MeV$, $v = 0.428195485c = 128{,}369{,}777\ m \cdot s^{-1}$

1.4 (a)	$at\ 1\ MeV, v = 0.046\ c = 1.38 \times 10^7\ m \cdot s^{-1}$	0.25 pts
	$at\ 10\ MeV, v = 0.145\ c = 4.34 \times 10^7\ m \cdot s^{-1}$	0.25 pts
	$at\ 100\ MeV, v = 0.428\ c = 1.28 \times 10^8\ m \cdot s^{-1}$	0.25 pts

1.4 (b)

Step 1: Calculate $F(\beta)$ using the Bethe equation: $\boldsymbol{F(\beta_v) = ln\frac{1.02 \times 10^6 \beta_v{}^2}{1-\beta_v{}^2} - \beta_v{}^2}$

At $\beta_v{}^2 = 0.046132147^2 = 0.002128175$, $F(\beta_v) = 7.682824962$

At $\beta_v{}^2 = 0.144844004^2 = 0.020979785$, $F(\beta_v) = 9.971340488$

At $\beta_v{}^2 = 0.428195485^2 = 0.183351373$, $F(\beta_v) = 12.158157270$

Step 2: Using the derived formula for mass stopping power:

$$\boldsymbol{\frac{S}{\rho} = -\frac{dE}{\rho dx} = \frac{0.170}{\rho \beta_v{}^2}[F(\beta_v) - 4.31]}$$

At $F(\beta_v) = 7.682824962$, $\frac{S}{\rho} = -\frac{dE}{\rho dx} = 269.423451250\ MeV \cdot cm^2 \cdot g^{-1} = 4.316648651 \times 10^{-12}\ J \cdot m^2 \cdot kg^{-1}$

At $F(\beta_v) = 9.971340488$, $\frac{S}{\rho} = -\frac{dE}{\rho dx} = 45.874057356\ MeV \cdot cm^2 \cdot g^{-1} = 7.349849721 \times 10^{-13}\ J \cdot m^2 \cdot kg^{-1}$

At $F(\beta_v) = 12.158157271$, $\frac{S}{\rho} = -\frac{dE}{\rho dx} = 7.276666183\ MeV \cdot cm^2 \cdot g^{-1} = 1.165852903 \times 10^{-13}\ J \cdot m^2 \cdot kg^{-1}$

1.4 (b)

$$at\ 1\ MeV, \frac{S}{\rho} = -\frac{dE}{\rho dx} = 269\ MeV \cdot cm^2 \cdot g^{-1}$$ 0.25 pts

$$at\ 10\ MeV, \frac{S}{\rho} = -\frac{dE}{\rho dx} = 45.9\ MeV \cdot cm^2 \cdot g^{-1}$$ 0.25 pts

$$at\ 100\ MeV, \frac{S}{\rho} = -\frac{dE}{\rho dx} = 7.28\ MeV \cdot cm^2 \cdot g^{-1}$$ 0.25 pts

Other acceptable answers:

$$at\ 1\ MeV, \frac{S}{\rho} = -\frac{dE}{\rho dx} = 4.32 \times 10^{-12}\ J \cdot m^2 \cdot kg^{-1}$$

$$at\ 10\ MeV, \frac{S}{\rho} = -\frac{dE}{\rho dx} = 7.35 \times 10^{-13}\ J \cdot m^2 \cdot kg^{-1}$$

$$at\ 100\ MeV, \frac{S}{\rho} = -\frac{dE}{\rho dx} = 1.17 \times 10^{-13}\ J \cdot m^2 \cdot kg^{-1}$$

1.4 (c)

Using the Bragg-Kleeman formula, the range R of proton in water is calculated given its kinetic energy.

$$\boldsymbol{R = N_R \times E_K{}^{\beta_e}}$$

where:
R is the range of a particle, in g·cm^{-2}
N_R is the proportionality factor (ICRU data for water, $N_R = 0.0023 \frac{g}{cm^2 \cdot MeV}$)
E_K is the kinetic energy of a particle
β_e is the exponent factor of the incident energy (ICRU data, β_e = 1.75)

Solving for R,

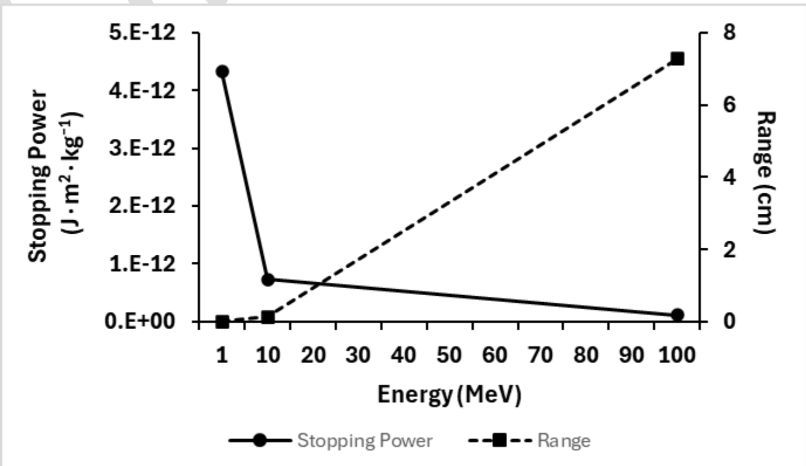
At $E_K = 1\ MeV,\ R = 0.0023\ g \cdot cm^{-2} = 0.0023\ cm$

At $E_K = 10\ MeV,\ R = 0.129338505\ g \cdot cm^{-2} = 0.129\ cm$

At $E_K = 100\ MeV,\ R = 7.273238618\ g \cdot cm^{-2} = 7.27\ cm$

1.4 (c)	$at\ 1\ MeV, R = 0.00230\ or\ 2.30 \times 10^{-3}\ \ cm$	0.25 pts
	$at\ 10\ MeV, R = 0.129\ or\ or\ 1.29 \times 10^{-1}\ cm$	0.25 pts
	$at\ 100\ MeV, R = 7.27\ cm$	0.25 pts
	Other acceptable answers:	
	$at\ 1\ MeV, R = 0.00230\ or\ 2.30 \times 10^{-3}\ g \cdot cm^{-2}$	
	$at\ 10\ MeV, R = 0.129\ or\ or\ 1.29 \times 10^{-1}\ g \cdot cm^{-2}$	
	$at\ 100\ MeV, R = 7.27\ g \cdot cm^{-2}$	

1.4 (d)

1.4 (d)		0.3 pts

The mass stopping power decreases, whereas the range of proton increases with increasing proton energy. **Note:** ACCEPT answer even if: (1) Unit of Energy is in joules (J). (2) Energy (1,10,100 MeV) is not graphed to the appropriate scale. (3) Log_{10} of energy values is used in the graph. (4) Obtained values (stopping power & range) are graphed with proton energy separately.	0.2 pts

1.5

Dose to the medium (water) can be calculated by the residual range method. After traveling 5.491 cm, the protons have enough energy to travel the residual range of:

$$\boldsymbol{R_{res}} = \boldsymbol{R_{CSDA}} - \boldsymbol{thickness} = 7.718\ g \cdot cm^{-2} - 5.491\ g \cdot cm^{-2} = 2.227\ g \cdot cm^{-2}$$

Table1: Proton Stopping Power and Range (in Water, Liquid)

Kinetic Energy (MeV)	S, total (MeV-cm^2 g^{-1})	Range, CSDA (g cm^{-2})
5.000E+01	1.245E+01	2.227E+00
5.500E+01	1.154E+01	2.644E+00
6.000E+01	1.078E+01	3.093E+00
6.500E+01	1.013E+01	3.572E+00
7.000E+01	9.559E+00	4.080E+00
7.500E+01	9.063E+00	4.618E+00
8.000E+01	8.625E+00	5.184E+00
8.500E+01	8.236E+00	5.777E+00
9.000E+01	7.888E+00	6.398E+00
9.500E+01	7.573E+00	7.045E+00
1.000E+02	7.289E+00	7.718E+00

Source: https://physics.nist.gov/cgi-bin/Star/ap_table-t.pl

The energy that has this range of 2.227 g/cm^2 is 50 MeV, which is the average energy at the exit. Thus, $E_{abs} = 100\ MeV - 50\ MeV = 50\ MeV$

Using the equation: $\boldsymbol{D} = \frac{\boldsymbol{E_{abs}}}{\boldsymbol{m}} = \frac{(\boldsymbol{dE/dx}) \times \boldsymbol{dx} \times \boldsymbol{N_p}}{\boldsymbol{\rho \times A \times dx}} = \boldsymbol{\Phi}\frac{\boldsymbol{dE}}{\boldsymbol{\rho dx}}$

$$D = \frac{(1 \times 10^9\, protons/cm^2)\,(50\; MeV/proton)}{(1\, g/cm^3)(5.491\; cm)} \times 10^6\; eV \times 1.60218 \times 10^{-19}\, J \times 10^3$$

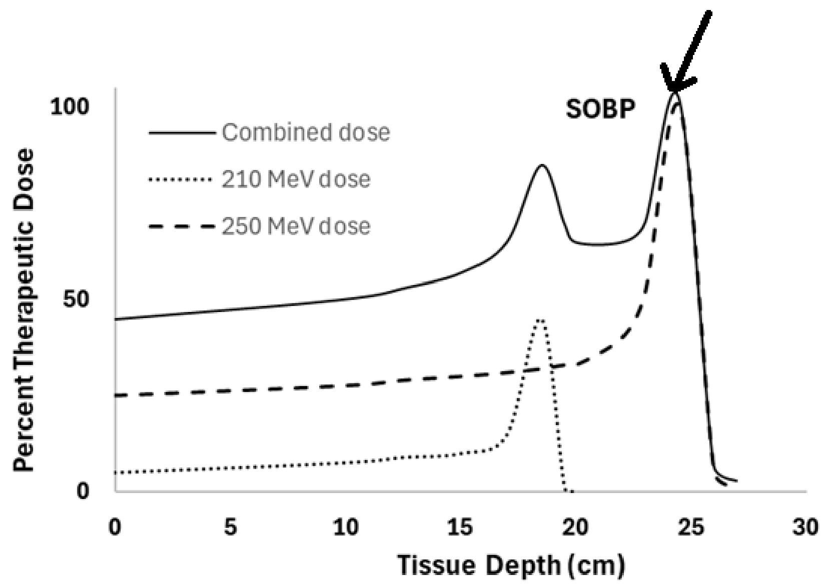
$$D = 1.458914588\, \frac{J}{kg} (or\; Gy)$$

1.5	$D = 1.46\, \frac{J}{kg}\; (or\; Gy)$ Other acceptable answer: $D = 9.11\, \times 10^9\, \frac{MeV}{g}$	0.75 pts

Part 2. Proton Therapy (5.0 pts)

2.1

2.1 (a) — 0.3 pts



2.1 (b)

Explanation: The rate of energy loss is proportional to the square of the particle charge and inversely proportional to the square of velocity.

2.1 (b)	FALSE	0.3 pts

2.1 (c)

Explanation: The relatively high exit dose from photon therapy limits the possibility of dose escalation or acceleration for tumors. In contrast, a proton beam is composed of charged particles (protons) with a well-defined range of penetration into tissue. Studies estimated proton fields could reduce approximately 50% of the irradiation dose to adjacent normal tissue compared with photon beams. Thus, proton therapy is ideal when organ preservation is a priority.

2.1 (c)	TRUE	0.3 pts

2.1 (d)	D. 235 MeV Other acceptable answers: D or 235 MeV	0.3 pts

2.2

Starting from Eq. [7]: $\boldsymbol{\beta_x D_x^2 + \alpha_x D_x - \alpha_p D_p - \beta_p D_p^2 = 0}$

where:

a = β_x

b = α_x

c = $-\alpha_p D_p - \beta_p D_p^2$

Use the quadratic formula to solve for $RBE\, or \left(\frac{D_x}{D_p}\right)$ given the ff. expressions:

$$\boldsymbol{RBE_{max} = \frac{\alpha_p}{\alpha_x}}$$

$$\boldsymbol{RBE_{min} = \sqrt{\frac{\beta_p}{\beta_x}}}$$

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$D_x = \frac{-\alpha_x + \sqrt{{\alpha_x}^2 - 4\beta_x(-\alpha_p D_p - \beta_p D_p^2)}}{2\beta_x}$$

$$RBE = \frac{D_x}{D_p} = \frac{-\alpha_x + \sqrt{{\alpha_x}^2 - 4\beta_x(-\alpha_p D_p - \beta_p D_p^2)}}{2\beta_x D_p}$$

$$RBE = \frac{D_x}{D_p} = \frac{-\frac{\alpha_x}{\beta_x} + \sqrt{\frac{\alpha_x^2}{\beta_x^2} + \frac{4\beta_x \alpha_p D_p}{\beta_x^2} + \frac{4\beta_x \beta_p D_p^2}{\beta_x^2}}}{2D_p}$$

$$RBE = \frac{D_x}{D_p} = \frac{-\left(\frac{\alpha}{\beta}\right)_x + \sqrt{\left(\frac{\alpha}{\beta}\right)_x^2 + \frac{4\beta_x(\alpha_x RBE_{max})D_p}{\beta_x^2} + \frac{4\beta_x(\beta_x RBE_{min}^2)D_p^2}{\beta_x^2}}}{2D_p}$$

$$RBE = \frac{D_x}{D_p} = \frac{\sqrt{\left(\frac{\alpha}{\beta}\right)_x^2 + 4D_p\left(\frac{\alpha}{\beta}\right)_x RBE_{max} + 4D_p^2 RBE_{min}^2} - \left(\frac{\alpha}{\beta}\right)_x}{2D_p}$$

2.2	$$RBE = \frac{D_x}{D_p} = \frac{\sqrt{\left(\frac{\alpha}{\beta}\right)_x^2 + 4D_p\left(\frac{\alpha}{\beta}\right)_x RBE_{max} + 4D_p^2 RBE_{min}^2} - \left(\frac{\alpha}{\beta}\right)_x}{2D_p}$$	1.0 pts

2.3 (a)

Solve for RBE_{max} and RBE_{min} using the provided equations below:

$$\boldsymbol{RBE_{max} = p_0 + \frac{p_1}{(\alpha/\beta)_x} LET_d}$$

$$\boldsymbol{RBE_{min} = p_2 + p_3 \sqrt{\left(\frac{\alpha}{\beta}\right)_x} LET_d}$$

where $p_{0\text{-}3}$ are the fit parameters for the LQ model.

$p_0 = 0.999064$

$p_1 = 0.35605\ Gy\ (KeV\ \mu m)^{-1}$

$p_2 = 1.1012$

$p_3 = 0.0038703\ Gy^{-1/2}\ (KeV\ \mu m)^{-1}$

$$\boldsymbol{RBE_{max} = 0.999064 + \frac{0.35605\ Gy\ KeV \mu m^{-1}}{(\alpha/\beta)_x} LET_d}$$

$$\boldsymbol{RBE_{min} = 1.1012 + 0.0038703\ Gy^{-1/2} KeV \mu m^{-1} \sqrt{\left(\frac{\alpha}{\beta}\right)_x} LET_d}$$

Cell lines	$(\alpha/\beta)_x$ Gy
HaCat	15.0
SKMel	3.0

LET_d	HaCat			SKMel		
	RBE_{max}	RBE_{min}	RBE_{2Gy}	RBE_{max}	RBE_{min}	RBE_{2Gy}
1.9	1.044164	1.129680	1.06	1.224562	1.113937	1.16
2.5	1.058406	1.138674	1.08	1.295772	1.117959	1.19
4.5	1.105879	1.168653	1.12	1.533139	1.131366	1.29

2.3 (a)

HaCat — 0.6 pts (0.2 pt each)

LET_d	RBE_{2Gy}
1.9	**1.06**
2.5	**1.08**
4.5	**1.12**

SKMel — 0.6 pts (0.2 pt each)

LET_d	RBE_{2Gy}
1.9	**1.16**
2.5	**1.19**
4.5	**1.29**

2.3 (b)

RBE increases with increasing LET_d. — 0.2 pts

Higher (or lower) RBE values can be expected for tissues with low (or high) $(\alpha/\beta)_x$ given similar LET_d values. — 0.2 pts

2.3 (c) Solve for D_x given the obtained values for RBE at $\boldsymbol{D_p = 2\ Gy}$.

$$\boldsymbol{D_x = RBE \times D_p}$$

2.3 (c)

HaCat — 0.6 pts (0.2 pt each)

LET_d	$D_x\ (Gy)$
1.9	**2.13**
2.5	**2.15**

4.5	**2.24**

SKMel

LET_d	D_x (Gy)
1.9	**2.32**
2.5	**2.38**
4.5	**2.57**

0.6 pts (0.2 pt each)

Q4 Mass and Abundance of Isotopes (10 pts)

Part 1. Basic operation of a Mass Spectrograph (2.1 pts)

1.1 Increase in kinetic energy:

$\Delta K = qV_a = (1.60218 \times 10^{-19}\text{C})(1.20 \times 10^{3}\text{V})$

$= 1.92 \times 10^{-16}\ \text{J}$

1.1	1.92×10^{-16} J Accept 1.91 – 1.93	0.3 pts

1.2. The velocity is determined obtained from the kinetic energy:

$K = \frac{1}{2}mv^2 \rightarrow v = \sqrt{\frac{2K}{m}}$

$v = \sqrt{\frac{2(1.923\times10^{-16}\ \text{J})}{12\times1.66057\times10^{-27}\text{kg}}} = 1.389 \times 10^{5}\ \text{m/s}$

1.2	1.40×10^{5} m/s Accept 1.3 – 1.5	0.4 pts

1.3 (a) Electric force acting on the ion is in the upward direction. The magnetic force on the ion must be in the downward direction (and equal to the electric force) for the ion to travel along a straight line. Therefore, *B1* must be in the *oz* direction.

1.3. (a)	oz direction	0.3 pts

1.3 (b) For the ions to travel along a straight line:

Force due to electric field ($F = qE$) = Force due to magnetic field ($F = qvB_1$

$qE = qvB_1$; since $E = \frac{V}{d}$, we get:

$\frac{V_s}{d} = vB_1, \qquad B_1 = \frac{V_s}{vd} = \frac{400\ \text{J/C}}{(140\times10^{3}\text{m/s})(2\times10^{-2}\text{m})} = 0.14\ \text{T}$

1.3 (b)	0.14 T Accept 0.13 – 0.15	0.7 pts

1.3 (c) According to the equations in part 1.3 (b), velocity depends only on the external electric field and the applied magnetic field in the velocity selector. Therefore, velocity of the doubly charged ions is also $140.0\ \mathrm{km/s}$.

1.3 (c)	$140.0\dfrac{\mathrm{km}}{\mathrm{s}}$ Accept 140 km/s also as a correct answer.	0.4 pts

Part 2. Motion of ions inside the ion separator (3.2 pts)

2.1 If the radius of the circular path of the ion inside the ion separator is R,

$$B_2 e v = \frac{mv^2}{R}$$

$$R = \frac{mv}{B_2 e} = \frac{(12 \times 1.66057 \times 10^{-27}\mathrm{kg})(140 \times 10^3 \mathrm{m/s})}{(0.030\ \mathrm{T})(1.60218 \times 10^{-19}\mathrm{C})} = 0.580\ \mathrm{m} = 58.0\ \mathrm{cm}$$

Thus, the ion hits the photographic plate at a distance of 116 cm from the center (*P*).

2.1	116 cm or 1.16 m Accept 1.15 m – 1.17 m as correct	0.7 pts

2.2 If x is the distance to the incident point of the ion from P on the photographic plate:

$$x = 2R = \frac{2mv}{B_2 e}$$

It is clear that the velocity v depends only on electric and magnetic fields applied in the velocity selector. Therefore, if the uncertainties related to electric field and magnetic fields are negligible,

$$(\Delta m)_{min} = \frac{B_2 e}{2v}(\Delta x) = \frac{(0.030\ \mathrm{T})(1.60218 \times 10^{-19}\mathrm{C})}{2\left(140.0 \times \frac{10^3\mathrm{m}}{\mathrm{s}}\right)}(10^{-3}\ \mathrm{m})$$

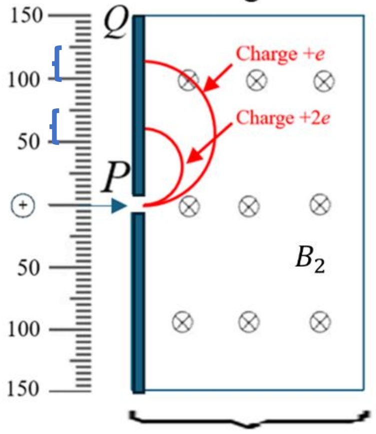
$$= 1.717 \times 10^{-29}\ \mathrm{kg}\left(\frac{1\mathrm{u}}{1.66057 \times 10^{-27}\mathrm{kg}}\right) = 0.01\ \mathrm{u}$$

2.2	0.01 u	1.0 pt

2.3 The ion beam produced in the ion source contains **both** singly and doubly ionized ^{12}C atoms. Their paths are shown below:

2.3

Figure 02



ion separator

For correct direction for both ions (with identification)	0.6 pts
For correct dimensions for both ions	0.6 pts

Note: Look for the following when awarding marks

1. Shape of the curves must be a semi-circle, approximately.
2. Acceptable range for the points of incidence are marked with { symbol. **Do not** award marks for **correct dimensions** if the points of incidence are outside.

Part 3: Mass spectrometers (1.6 pts)

3.1 We can use the same equation $x = 2R = \frac{2mv}{B_2 e}$ obtained in part **2.2**, but here we have fixed x and variable B_2. We get the expression for m from the above expression:

$$m = \frac{B_2 ex}{2v}$$

For $B_2 = 20\ \text{mT}$:

$$m_{\min} = \frac{(20\times10^{-3}\ \text{T})(1.60218\times10^{-19}\text{C})(0.8\text{m})}{2(140\times10^3 \text{m}.s^{-1})} = 9.15531\times10^{-27}\ \text{kg}\left(\frac{1\text{u}}{1.66057\times10^{-27}\text{kg}}\right) = 5.51\ \text{u}$$

Similarly, for $B_2 = 320\ \text{mT}$;

$$m_{\max} = \frac{(320\times10^{-3}\ \text{T})(1.60218\times10^{-19}\text{C})(0.8\text{m})}{2(140\times10^3 \text{m}.s^{-1})} = 1.46485\times10^{-25}\ \text{kg}\left(\frac{1\text{u}}{1.66057\times10^{-27}\text{kg}}\right) = 88.2\ \text{u}$$

Therefore, elements from Li (atomic mass~6 u) to Zr (atomic mass~87 u) can be analyzed with this spectrometer.

[Note: Identification of elements cannot be done exactly because of the overlapping of the atomic masses of the isotopes of different elements]

3.1	$m_{min} = 5.51$ u Accept 5.50-5.52 $m_{max} = 88.2$ u Accept 88.1 – 88.3	0.5 pts 0.5 pts

3.2 Because the electric current represents the total charge that passes through the exit slit per unit time,

$$\frac{Abundance\ of\ ^{12}\mathrm{C}}{Abundance\ of\ ^{13}\mathrm{C}} = \frac{16.5}{185\times10^{-3}} = 89.19$$

3.2	89.19 or 89.2 No range can be given because this is a simple ratio.	0.6 pts

Part 4: Determining the age of rock samples (3.4 pts)

4.1 (a) The relationship between $N_P(t_0)$, $N_D(t_0)$, $N_P(t_1)$ and $N_D(t_1)$ is expressed below:

4.1 (a)	$N_P(t_0)e^{-\lambda(t_1-t_0)} + N_D(t_1) = N_P(t_0) + N_D(t_0)$ (or $N_D(t_1) = N_P(t_0)[1 - e^{-\lambda(t_1-t_0)}] + N_D(t_0)$ or any other correct form)	0.4 pts

4.1 (b) From the radioactive decay law $N_P(t_1) = N_P(t_0)e^{-\lambda(t_1-t_0)}$ and substituting this in the equation obtained in 4.1 (a):

$$N_P(t_1) + N_D(t_1) = N_P(t_1)e^{\lambda(t_1-t_0)} + N_D(t_0)$$

$$N_D(t_1) = N_P(t_1)[e^{\lambda(t_1-t_0)} - 1] + N_D(t_0)$$

By dividing the entire equation by $N_{D_s}(t_1)$,

$$\frac{N_D(t_1)}{N_{D_s}(t_1)} = \frac{N_P(t_1)}{N_{D_s}(t_1)}[e^{\lambda(t_1-t_0)} - 1] + \frac{N_D(t_0)}{N_{D_s}(t_0)}$$

Therefore, $G = [e^{\lambda(t_1-t_0)} - 1]$

Note that because the element D_s is stable $N_{D_s}(t_1) = N_{D_s}(t_0)$.

4.1 (b)	$G = [e^{\lambda(t_1-t_0)} - 1]$	0.7 pts

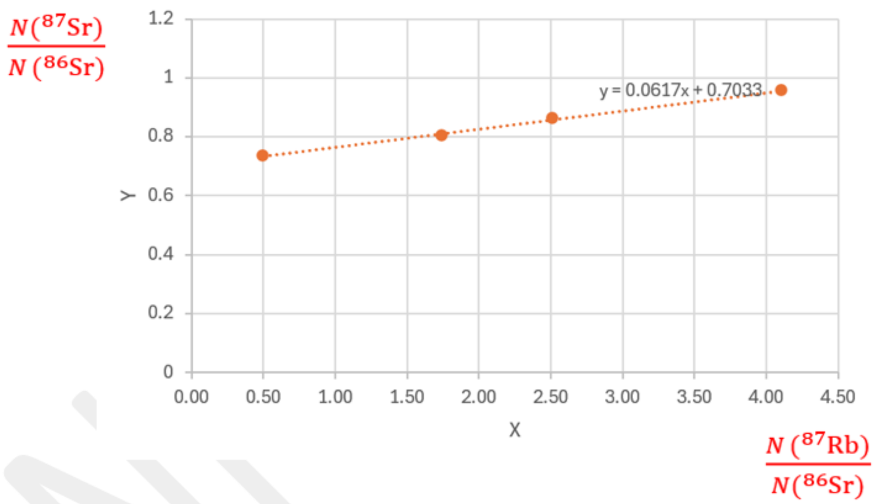
4.2 (a) Based on the current measurements given in Table 01, the entries in Table 02 are determined as follows:

4.2 (a) 0.8 pts *(0.1 pt for each entry)*

Table 1. Nuclide ratios for the rock samples

Rock Sample	$\frac{N(^{87}\mathrm{Rb})}{N(^{86}\mathrm{Sr})}$	$\frac{N(^{87}\mathrm{Sr})}{N(^{86}\mathrm{Sr})}$
1	2.51	0.864
2	0.50	0.736
3	1.74	0.804
4	4.11	0.956

4.2 (b)



For the selection of the correct axes 0.2 pts

For the correct marking of the points (0.1/each) 0.4 pts

4.2 (c) Gradient = 0.0617 0.3 pts

Accept the values in the range 0.060 to 0.064

4.2 (d) From the given half-life, $\lambda = \frac{0.693}{4.8\times10^{10}}\ yr^{-1} = 1.44\times10^{-11}\ yr^{-1}$

Gradient $= \left[e^{\lambda(t_1-t_0)} - 1\right] = 0.0617$

$$e^{\lambda(t_1-t_0)} = 1.0617$$

Therefore age of the rock samples $=(t_1 - t_0) = \frac{\ln(1.0617\}}{1.44\times10^{-11}}\ y$

$$= 4.15\times10^{9}\ years \sim 4.2\ billion\ years$$

$$= 4.15\times10^{9}\ years \sim 4.2\ billion\ years$$

4.2 (d)	$4.15 \times 10^9\ years \sim 4.2\ billion\ years$ Acceptable range 4.0 to 4.3 billion years. Warning: There is a possibility that students writing the answer here without calculating. It is impossible that he/she gets the correct answer if his/her answer to 4.2(c) is wrong. Therefore, **do not award marks for this part if his/her answer to 4.2(c) is wrong.**	0.6 pts

Q5 ADVANTAGES OF USING LOW ENRICHED URANIUM IN NUCLEAR REACTORS (10 pts)

Note: Answer key demonstrates the solution process from formula derivation up to substitution of given values. For brevity, the results of intermediate calculations are presented in an arbitrary number of significant figures.

For the boxed results containing numeric answers, three values are given: **low value**, **exact value**, **high value**. Exact value is the result obtained when computations are performed starting from the problem given without rounding-off until final answer. The low and high value corresponds to $\pm 5\%$ of the exact value. **Numerical results ranging from the low value up to the high value are considered correct regardless of how many significant figures are presented.** This was done to accommodate examinee solutions implementing the correct process but failed to follow the instruction not to truncate in intermediate calculations.

Part 1. Uranium Enrichment (3.5 pts)

1.1a

We first get the conservation of the mass of the total Uranium content, given the mass of the product M_P, mass of the feed M_F, and the mass of the tail M_T. We know that the feed is put into the enrichment plant and after the whole process, we have the enriched product and the depleted uranium (tail). We can then write the first conservation of mass equation as

$$M_F = M_P + M_T \tag{1.1}$$

Next is that the total U-235 content is the same. As defined, x is the enrichment of U-235 in weight fraction, this means that the U-235 content in the feed is given by $x_F M_F$, and same for the enriched product and tail. We can now write the conservation of mass of the U-235 as

$$x_F M_F = x_P M_P + x_T M_T \tag{1.2}$$

1.1a	(i) $M_F = M_P + M_T$	0.05 pt
	(ii) $x_F M_F = x_P M_P + x_T M_T$	0.05 pt

1.1b

Using the results of 1.1a, we can isolate M_T

$$M_F = M_P + M_T \rightarrow M_T = M_F - M_P \quad (1.3)$$

and

$$x_{FM_F} = x_P M_P + x_T M_T \rightarrow x_T M_T = x_F M_F - x_P M_P \quad (1.4)$$

$$x_T M_T = x_F M_F - x_P M_P \rightarrow M_T = \frac{x_F M_F - x_P M_P}{x_T} \quad (1.5)$$

Using equations (1.3) and (1.5), we get

$$M_F - M_P = \frac{x_F M_F - x_P M_P}{x_T} \quad (1.6)$$

$$x_T M_F - x_T M_P = x_F M_F - x_P M_P \quad (1.7)$$

$$x_T M_F - x_F M_F = x_T M_P - x_P M_P \quad (1.8)$$

$$(x_T - x_F) M_F = (x_T - x_P) M_P \quad (1.9)$$

Finally, isolating either M_P or M_F, we get,

$$M_F = \frac{x_T - x_P}{x_T - x_F} M_P = \frac{x_P - x_T}{x_F - x_T} M_P \quad (1.10)$$

or

$$M_P = \frac{x_T - x_F}{x_T - x_P} M_F = \frac{x_F - x_T}{x_P - x_T} M_F \quad (1.11)$$

1.1b	$M_F = \frac{x_T - x_P}{x_T - x_F} M_P$ $M_F = \frac{x_P - x_T}{x_F - x_T} M_P$ $M_P = \frac{x_T - x_F}{x_T - x_P} M_F$ $M_P = \frac{x_F - x_T}{x_P - x_T} M_F$ Any of the 4 possible answers will merit full credit (0.3 pt)	0.3 pt

1.2

Using the result from 1.1b,

$$M_{235} = x_P M_P \rightarrow M_P = \frac{M_{235}}{x_P} \quad (1.12)$$

$$M_F = \frac{x_P - x_T}{x_F - x_T}\frac{M_{235}}{x_P} \quad (1.13)$$

We are given the values of $x_T = 0.00200$ and $x_F = 0.00720$ and by substituting these values, we get,

$$M_F = \frac{x_P - 0.00200}{0.00720 - 0.00200}\frac{M_{235}}{x_P} = \frac{x_P - 0.00200}{0.00520}\frac{M_{235}}{x_P} \quad (1.14)$$

For a), $x_P = 0.03$,

$$M_F = \frac{0.03000 - 0.00200}{0.00520}\frac{M_{235}}{0.03000} \quad (1.15)$$

$$M_F = 179.48718\, M_{235} \quad (1.16)$$

For b), $x_P = 0.197$,

$$M_F = \frac{0.19700 - 0.00200}{0.00520}\frac{M_{235}}{0.19700} \quad (1.17)$$

$$M_F = 190.35533\, M_{235} \quad (1.18)$$

1.2a (0.5 pt)

Low Value	Exact Value	High Value
$170.5\ M_{235}$	$179.5\ M_{235}$	$188.5\ M_{235}$

Or

Low Value	Exact Value	High Value
170.5	179.5	188.5

1.2b (0.5 pt)

Low Value	Exact Value	High Value
$180.8\ M_{235}$	$190.4\ M_{235}$	$199.9\ M_{235}$

Or

Low Value	Exact Value	High Value
180.8	190.4	199.9

1.3a

We are given the values $x_T = 0.00200$, $x_F = 0.00720$, and $M_F = 1000$ kg. We are also given the follow values for x_P: 1.0%, 3.0%, 10.0%, 20.0%, 50.0%, 90.0%. We first calculate for the corresponding mass of the product M_P using the result of problem 1.1b.

x_P	M_P	x_P	M_P
1.0%	650.00000	20.0%	26.26263

3.0%	185.41729	50.0%	10.44177
10.0%	53.06112	90.0%	5.79065

Next step is to calculate for the corresponding values from the value function.

$$V(x_T) = V(0.00200) = 6.18776 \quad (1.19)$$

$$V(x_F) = V(0.00720) = 4.85551 \quad (1.20)$$

x_P	$V(x_P)$	x_P	$V(x_P)$
1.0%	4.50322	20.0%	0.83178
3.0%	3.26753	50.0%	0.00000
10.0%	1.75778	90.0%	1.75778

To get the final answers, we use equation 1 of the problem to get the value for each enrichment value for the product.

$$SWU(1.0\%) = M_P[V(1.0\%) - V(0.002)] - M_F[V(0.0072) - V(0.002)] \quad (1.21)$$

$$SWU(1.0\%) = 650[V(1.0\%) - V(0.002)] - 1000[V(0.0072) - V(0.002)] \quad (1.22)$$

$$SWU(1.0\%) = 650[4.50322 - 4.85551] - 1000[4.85551 - 6.18776] \quad (1.23)$$

$$SWU(1.0\%) = -1094.95100 - (-1332.25000) = 237.29900 \quad (1.24)$$

Following the same process for the other enrichment percentage, we get,

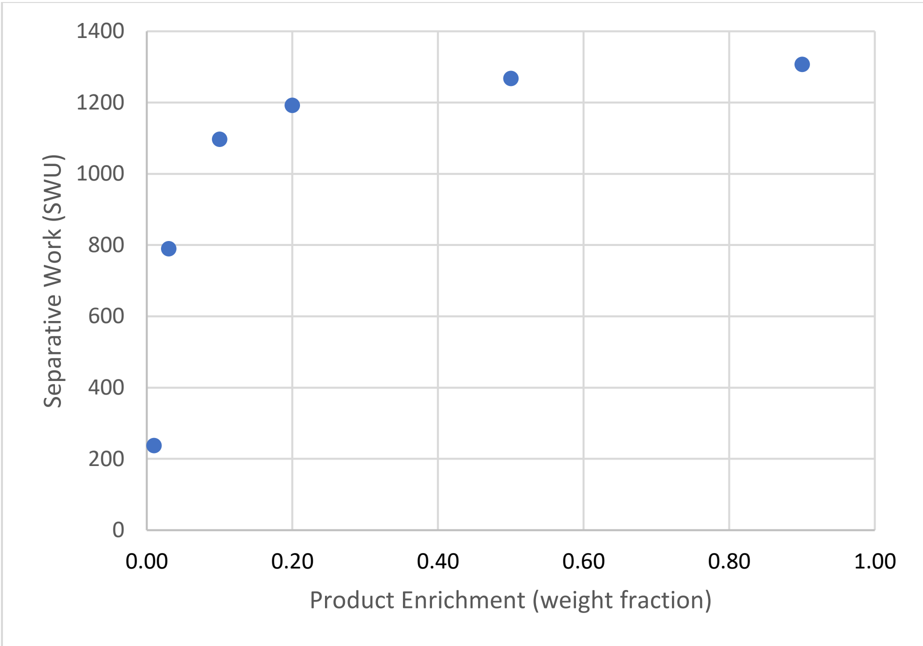
1.3a — 1.2 pt; 0.2 pt per answer for a given x_P

x_P	SWU		
	Low Value	Exact Value	High Value
1.0%	225.4	237.3	249.2
3.0%	750.4	789.9	829.4
10.0%	1042	1097	1152
20.0%	1132	1192	1251
50.0%	1204	1268	1331
90.0%	1241	1307	1372

1.3b

We take the values from problem **1.3a** and plot the value of the separative work units by their enrichment.

1.3b — 0.6 pt



Grading Rubric:

Case for 0.6 pt:

- All answers to problem 1.3a are correct and data points placed appropriately.

Case for 0.4 pt:

- Correct trend of steep climb from 0 to 0.2 weight fraction then gradual increase thereafter. Acceptable even without dots/markers. Acceptable even if the y-axis range is missing or incorrect.

Case for 0.2 pt:

- Examinee wrote values in y-axis regardless of the range .

1.3c	Difficult for enrichment plants to achieve enrichment from a lower percentage up to the 20% mark. After the 20% mark, we	0.3 pt

	can see that a lesser amount of work is needed to get to the higher enrichment values. Or More work to enrich from low percentage up to 20% than increasing beyond 20% to higher enrichment level. Grading Rubric: Full credit is given when any of the following thoughts are expressed: • More effort required to enrich from low value up to 0.2 weight fraction (20% enrichment) • Less effort required to enrich above 0.2 weight fraction (20% enrichment)	

Part 2. Energy from ^{235}U and ^{238}U (4.0 pts)

2.1

Given the total net power of the nuclear reactors of the hypothetical region:

$$\text{Total Net Power} = 5708\ MW$$

Let n be the number of fissions per second. The recoverable energy that is eventually converted into electricity can be calculated as follows, noting the 5% of the total energy that is carried away by neutrinos, and the 30% thermal conversion efficiency:

$$\begin{aligned}\text{So available thermal energy per second} \\ &= \text{energy released per fission} \times \text{recoverable fraction} \times \mathrm{n} \\ &\times \text{Elementary charge} \\ &= 200 \times (1-0.05) \times \mathrm{n} \times 1.602 \times 10^{-13}\ \mathrm{J}\end{aligned}$$

Electrical power can be obtained noting the 30% thermal conversion efficiency of the nuclear power plants:

$$\begin{aligned}\text{Total Net Power} &= 200 \times (1-0.05) \times \mathrm{n} \times 1.602 \times 10^{-13} \times 0.3 \\ &= 9.1314 \times 10^{-12} \times \mathrm{n}\end{aligned}$$

$$9.1314 \times 10^{-12} \times \mathrm{n} = 5.708 \times 10^{9}\ \mathrm{W}$$

$$n = 6.250 \times 10^{20} \frac{\text{fissions}}{\text{sec}}$$

Total number of neutrons produced in a year can be obtained from the neutron multiplicity ($\nu_f = 2.6$), and the number of fission events in a year.

$$\text{Total No of Neutrons produced in one year}$$
$$= 6.250 \times 10^{20} \frac{\text{fissions}}{\text{sec}} \times 365 \frac{\text{days}}{\text{year}} \times 86400 \frac{\text{sec}}{\text{days}} \times 2.6$$
$$= 6.250 \times 10^{20} \frac{\text{fissions}}{\text{sec}} \times 3.154 \times 10^{7} \frac{\text{sec}}{\text{year}} \times 2.6$$
$$= 5.125 \times 10^{28}$$

Noting the problem given that 20% of the fission neutrons are used to convert ^{238}U to ^{239}Pu:

$$\text{Neutrons absorbed in } ^{238}\text{U in resonance} = 5.125 \times 10^{28} \times 0.2 = 1.025 \times 10^{28}$$

The ^{239}Pu mass can be estimated using the assumption that the atomic mass value is approximated by the mass number:

$$\text{Total Plutonium } (^{239}\text{Pu}) \text{ Produced in one year} = \frac{1.025 \times 10^{28}}{6.022 \times 10^{23}} \times 239$$

$$\text{Total Plutonium } (^{239}\text{Pu}) \text{ Produced in one year} = 4068\text{kg}$$

2.1 0.6 pt

Low Value	Exact Value	High Value
3864 kg	4068 kg	4271 kg

2.2

2.2a

Considering the contribution of ^{239}Pu and ^{238}U, only 60% of the fissions are with ^{235}U:

$$\text{No of fissions in } ^{235}\text{U during one year} = \text{fraction of fission with } ^{235}\text{U} \times n \times \text{seconds per year}$$
$$\text{No of fissions in } ^{235}\text{U during one year} = 0.6 \times 6.2507 \times 10^{20} \frac{\text{fissions}}{\text{sec}} \times 3.154 \times 10^{7} \frac{\text{sec}}{\text{year}}$$
$$= 1.183 \times 10^{28} \text{ fissions}$$

^{235}U mass can be estimated using the assumption that the atomic mass value is approximated by the mass number:

$$^{235}\text{U consumed in fission} = \frac{\text{\# of fissions}}{\text{Avogadros constant}} \times {}^{235}\text{U atomic mass}$$
$$= \frac{1.183 \times 10^{28}}{6.022 \times 10^{23} \frac{1}{mol}} \times 235 \text{ grams/mol} = 4615 \text{ kg}$$

2.2a 0.6 pt

Low Value	Exact Value	High Value
4384 kg	4615 kg	4846 kg

2.2b

Considering that not all neutron absorptions by ^{235}U lead to fission, there is a need to account for the probability for fission upon neutron absorption given by the ratio of σ_f and σ_a for the specified nuclide.

$$\text{Fraction of thermal neutron inducing fission in } {}^{235}_{92}\text{U} = \frac{\sigma_f}{\sigma_a} = \frac{582.2}{680.8} = 0.8552$$

The amount of ^{235}U consumed can be obtained by dividing with the probability for fission

$$\text{Total } {}^{235}\text{U consumed} = \frac{4615}{0.8552} = 5396 \text{ kg}$$

2.2b 0.6 pt

Low Value	Exact Value	High Value
5127 kg	5396 kg	5666 kg

2.2c

The mass of feed material needed can calculated using the formula obtained in part 1.2

$$M_F = \frac{x_P - x_T}{x_F - x_T}\frac{M_{235}}{x_P}$$

with $x_T = 0.00200$, $x_F = 0.00720$, and $x_P = 0.035$, and M_{235} being the ^{235}U consumed as calculated in part 2.2b

$$\text{Total natural uranium used } = \left(\frac{0.035 - 0.002}{0.0072 - 0.002}\right)\frac{5396}{0.035} = 9.785 \times 10^5 \text{ kg}$$

2.2c 0.8 pt

Low Value	Exact Value	High Value
9.295×10^5 kg	9.785×10^5 kg	1.027×10^6 kg

Or

Low Value	Exact Value	High Value
929,500 kg	978,500 kg	1,027,000 kg

2.2d

Comparison to the Total mass of natural uranium used when considering only fissions of ^{235}U (Table 2), direct and indirect fissions in ^{238}U due to fertile conversion have saved:

$$1.631 \times 10^6 - 9.785 \times 10^5 = 6.525 \times 10^5 \text{ kg}$$

2.2d 0.2 pt

Low Value	Exact Value	High Value
6.199×10^5 kg	6.525×10^5 kg	6.852×10^5 kg

Or

Low Value	Exact Value	High Value
619,900 kg	652,500 kg	685,200 kg

2.3

Mass of ^{236}U is calculated from the difference of consumed and fissioned ^{235}U:

$$^{236}\text{U produced in one year} = 5396 - 4615 = 781.4 \text{ kg}$$

An alternative way to estimate the ^{236}U produced is via the capture-to-fission ratio:

$$\alpha = \frac{\sigma_\gamma}{\sigma_f} = \frac{680.8 - 582.2}{582.2} = 0.17$$

Mass of ^{236}U calculated from the capture-to-fission ratio is:

$$^{236}\text{U produced in one year} = \frac{^{235}\text{U(n}, \gamma) \text{ per year}}{6.02214 \times 10^{23}} \times 236 = \frac{^{235}\text{U(n, f) per year} \times \alpha}{6.02214 \times 10^{23}} \times 236$$
$$= \frac{1.183 \times 10^{28} \times 0.17}{6.02214 \times 10^{23}} \times 236 = 784.9 \text{ kg}$$

^{239}Pu contributes to 35% of the total number of fission events, therefore the mass of ^{239}Pu fissioned can be evaluated as follows:

$$\text{No of fissions in } ^{239}\text{Pu in one year}$$
$$= \text{fraction of fission with } ^{239}\text{Pu} \times n \times \text{seconds per year}$$
$$= 0.35 \times 6.250 \times 10^{20} \frac{\text{fissions}}{\text{sec}} \times 3.154 \times 10^{7} \frac{\text{sec}}{\text{year}}$$
$$= 6.899 \times 10^{27} \text{ fissions}$$

^{239}Pu mass can be estimated using the assumption that the atomic mass value is approximated by the mass number:

$$^{239}\text{Pu fissioned} = \frac{6.899 \times 10^{27}}{6.02214 \times 10^{23}} \times 239 \text{ grams} = 2738 \text{ kg}$$

Similar with ^{235}U not all absorptions in ^{239}Pu leads to fission, therefore to obtain the amount of ^{239}Pu consumed in the reactor, the probability for fission evaluated as follows must be accounted:

$$\text{Fraction of thermal neutron inducing fission in } ^{239}_{94}\text{Pu} = \frac{\sigma_f}{\sigma_a} = \frac{742.5}{1011.3} = 0.7342$$

$$\text{Total } ^{239}\text{Pu consumed} = \frac{2738}{0.7342} = 3729 \text{ kg}$$

Mass of ^{240}Pu produced is the difference between consumption and fission of ^{239}Pu since we assumed that the only competing reaction with fission is radiative capture:

$$^{240}\text{Pu produced in fuel after one year} = 3729 - 2738 = 991.2 \text{ kg}$$

An alternative way to estimate the ^{240}Pu produced is via the capture-to-fission ratio:

$$\alpha = \frac{\sigma_\gamma}{\sigma_f} = \frac{1011.3 - 742.5}{742.5} = 0.36$$

$$^{240}\text{Pu produced in one year} = \frac{^{239}\text{Pu}(n,\gamma) \text{ per year}}{6.02214 \times 10^{23}} \times 240 = \frac{^{239}\text{Pu}(n,f) \text{ per year} \times \alpha}{6.02214 \times 10^{23}} \times 240$$
$$= \frac{6.899 \times 10^{27} \times 0.36}{6.02214 \times 10^{23}} \times 240 = 995.3 \text{ kg}$$

2.3 ^{236}U — 0.4 pt

0.8 pt

Low Value	Exact Value	High Value
742.3 kg	781.4 kg	820.5 kg

Or

Low Value	Exact Value	High Value
745.7 kg	784.9 kg	824.2 kg

^{240}Pu

Low Value	Exact Value	High Value
941.6 kg	991.2 kg	1040.8 kg

Or

Low Value	Exact Value	High Value
945.6 kg	995.3 kg	1045.1 kg

Part 3. Criticality Accident (2.5 pts)

3.1

The energy yield can be evaluated given the energy released per fission event and the conversion relation to kg of TNT.

$$E = F_{yield} \cdot \epsilon = 6 \times 10^{16}[\text{fissions}] \cdot 200 \left[\frac{\text{MeV}}{\text{fission}}\right]$$
$$= 1.20 \times 10^{19}[\text{MeV}] \cdot 1 \times 10^{6} \left[\frac{\text{eV}}{\text{MeV}}\right] \cdot 1.602 \times 10^{-19} \left[\frac{\text{J}}{\text{eV}}\right]$$
$$\cdot \; 2.390 \times 10^{-7} \; [\text{kg of TNT}]$$

3.1 0.5 pt

Low Value	Exact Value	High Value
0.4365 kg TNT	0.4595 kg TNT	0.4825 kg TNT

3.2

$$E_{gamma} = C_{\gamma} \cdot \frac{F_{yield} \cdot \bar{\nu}_{\gamma}}{4\pi r^2} = 3.92[\text{pSv} \cdot \text{cm}^2] \cdot 1 \times 10^{-12} \left[\frac{\text{Sv}}{\text{pSv}}\right] \cdot \frac{6 \times 10^{16}[\text{fission}] \cdot 8.58 \left[\frac{\gamma}{\text{fission}}\right]}{4\pi(1.7 \times 10^2[\text{cm}^2])^2}$$

3.2 1.0 pt

Low Value	Exact Value	High Value
5.279 Sv	5.557 Sv	5.835 Sv

3.3

$$E_{neutrons} = C_n \cdot \frac{F_{yield} \cdot \bar{\nu}_n}{4\pi r^2} = 407[pSv \cdot cm^2] \cdot 1 \times 10^{-12}\left[\frac{Sv}{pSv}\right] \cdot \frac{6 \times 10^{16}[fission] \cdot 2.80\left[\frac{n}{fission}\right]}{4\pi(1.7 \times 10^2[cm^2])^2}$$
$$= 188.3 \ Sv$$

$$E_{total} = E_{neutrons} + E_{gamma} = 5.557 + 193.8$$

3.3

$E_{neutrons}$

0.6 pt
0.3 pt
0.1 pt

Low Value	Exact Value	High Value
178.9 Sv	188.3 Sv	197.7 Sv

E_{total}

Low Value	Exact Value	High Value
184.1 Sv	193.8 Sv	203.5 Sv

Which type of ionizing Radiation:

Neutron or Neutrons